COMPOSITIONAL AND INCREMENTAL MODELING AND ANALYSIS FOR
HIGH-CONFIDENCE DISTRIBUTED EMBEDDED CONTROL SYSTEMS

By

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DEDICATION

To God, the Father of us all, for helping me find insight to solve problems, for giving me courage to face fears and difficulties, and for putting wonderful people all along this journey. To Jesus Christ, the Son of God, for continuing to drive darkness from my life, replacing it with the light of peace, truth, joy, and love.

To Rebekah, my dearest companion, for many years of sacrifice and patience. This would never have happened without you. Thank you for never giving up on me. To my wonderful and patient children, for their encouragement and prayers.

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CHAPTER I

PROBLEMS IN MODEL-BASED DESIGN AND ANALYSIS FOR
HIGH-CONFIDENCE DISTRIBUTED EMBEDDED SYSTEMS

High confidence embedded control system software designs often require formal analyses to ensure design correctness. Detailed models of system behavior cover numerous design concerns, such as controller stability, timing requirements, fault tolerance, and deadlock freedom. Models for each of these domains must together provide a consistent and faithful representation of the potential problems an operational system would face. This poses challenges for structural representation of models that can integrate software design details between components and across design domains, as commonly components and design aspects are tightly coupled.

Coupling between separately designed components and modules can prevent model analyses from scaling well to large designs. Coupling also occurs within individual systems and components between behaviors represented by different design concerns (as represented by the layers shown in Fig. 1) as different aspects of the design constrain design structures and parameters in different ways. These complications combine with other factors to increase the difficulty of system integration. Integration difficulties are well-documented for embedded systems [3], and for software projects in general [4].

As a simple example from the distributed embedded control systems domain, schedulability, deadlock-freedom, and stability are three different notions of correctness that must be satisfied for virtually any distributed real-time control system design. All three of these conditions can depend on the frequencies at which real-time tasks are run, but each design concern constrains the frequencies in a different way. For example, increasing sampling frequency can increase the stability of a control loop, but which could make scheduling requirements difficult or impossible to satisfy. Extending the same example, specifying additional timing constraints and dependencies between tasks in a real-time system may improve end-to-end latency, but increase the risk of deadlock. We call this interaction of constraints between disparate design concerns vertical coupling.
A typical example of horizontal coupling (i.e. between components in a design, and within a single design concern) is the problem of end-to-end latency requirements. The latency incurred between a change in the physical environment and the control system’s response to that change is often a critical matter. Numerous sensing, computation, data communication, and actuation elements may lie on a timing-critical data dependency path between the initial sensing event and the final actuation event. These elements usually share the same computing and communication resources, so tightening one latency requirement to meet performance goals can easily render other seemingly unrelated requirements infeasible. Often horizontal coupling is implicit – we usually specify component parameters separately, resulting in adverse performance changes at the system level due to unanticipated coupling.
Our solutions to the larger coupling problems in high-confidence embedded systems design revolve around three main techniques:

1. **Model-based design and analysis**: Model-integrated computing [5] and related approaches can prevent numerous structural and conceptual errors by encoding correctness concepts into Domain-Specific Modeling Languages (DSMLs) used for design, and by resolving and encoding relationships between details in different design aspects as language structures and constraints. Inasmuch as system behaviors and behavioral notions of correctness can be encoded in a modeling language and efficiently analyzed, these notions of correctness can be addressed by the structure of design models [6].

2. **Vertical decoupling**: Several research efforts consider the problem of decoupling between design domains – within a design we want to increase behavioral independence of elements of the system with respect to a particular property so that design elements in different aspects can be specified more or less independently. Examples include the Time-Triggered architecture[7], which decouples functional specifications from timing specifications by providing a set of protocols to guarantee timing determinism and fault tolerance – protecting requirements in both design aspects. Synchronous execution models reduce deadlock-freedom and decidability of correctness properties to constraints on the structure and parameters of the design[8, 9]. Passive control theory guarantees stability of control loops through the proper interconnection of passive components to maintain passivity at the system level [10, 11], and decreases the destabilizing effects of sampling variance and time delays due to hardware [12, 13].

3. **Horizontal decoupling**: In model analysis we aim to exploit the structure of the model to increase the scalability of analyses for that model. Two general techniques for addressing scalability problems are compositional analysis and incremental analysis.

   (a) Compositional analysis is a structural property of a formal analysis method (and compatible models) allowing the partition of a design into components. As analysis proceeds we establish correctness properties first for the individual components and second for the compositions of components using formal models of their interactions. Scalability
comes from successive analysis and combination of small component models, as opposed to analyzing a single, large all-inclusive model.

(b) Incremental analysis relies on information stored in the model regarding previous analyses. The idea is to isolate the effects of model changes (i.e. adding new components, removing components, or modifying existing components) on the results of previous analyses. If done efficiently, analysis must only be computed for components in the vicinity of the changed components, where “vicinity” can be defined in a number of ways.

Cheaper analysis can yield significant cost reductions over the lifespan of a high-confidence control software development project. Incremental analysis can further reduce costs by allowing rapid redesigns when features are added to a deployed design.

**Specific Problems**

1. Continuous-time feedback control, embedded computer software, and distributed computing hardware design domains are highly specialized and often conceptually incompatible. Sharing model artifacts between designers in different domains can lead to inconsistency problems in software implementations or other engineering artifacts due to incomplete or faulty understanding of design issues. These inconsistencies can seriously impact the soundness of model analyses, and can hide design defects. Current state of the art resolves these problems by reviewing many of the details in meetings and personal discussions. In the worst cases serious incompatibilities are not discovered until very late in the development cycle, leading to project overruns and cancellations.

2. Controller properties which are verified using simulation models may no longer be valid when the design becomes software in a distributed processing network. Scheduling jitter, data communication delays, and precision loss due to data quantization are all examples of effects that contribute to instability and performance loss in controller software. Currently control designers use conservative performance margins to avoid rework when performance is lost due to deployment on a digital platform.
3. Long design, analysis, development, deployment, and test cycles limit the amount of iterative rework that can be done to get a correct design. Currently high-confidence design requires both long schedules and high costs. Lack of scalable formal model analysis methods is a significant factor preventing rapid design evaluations.

4. Automating steps in different design and analysis domains for the same models and tools requires a consistent view of inferred model relationships across multiple design domains. If integrated tools have different views of the model semantics, then their analyses are not valid when the results are integrated into the same design.

**Contributions**

In particular, we propose the following contributions toward solutions of the problems described above:

1. **Model Integration of High-Confidence Design Tools:** We have created a DSML for modeling and generating software implementations of distributed control systems, the Embedded Systems Modeling Language (ESMoL). ESMoL includes aspects for functional modeling, execution platforms, and mapping of functional blocks to execution platforms. The language also includes appropriate parameters to capture timing behavior.

2. **Extensible Language Interpreter Framework:** We use a two-stage interpreter development framework to isolate model interpreter code from the details of the front-end ESMoL modeling language as we experiment with language design. The first stage transforms ESMoL models to models in a language called ESMoL-Abstract, resolving inferred model relations. The second stage interpreters create analysis models, simulations, and platform-specific code. We aim to give all of the model interpreters a single, consistent view of model details for analysis and generation.

3. **Integrated Incremental Cycle Analysis:** The ESMoL tool suite includes an analyzer that checks for delay-free loops in the assembled dataflow models.
4. **Incremental Task Graph Schedule Calculation**: We present a conceptual discussion of an incremental method for calculating task graph schedules.

**Dissertation Organization**

- Chapter II discusses literature in the field that relates to our chosen solution methods.

- Chapter III discusses the design philosophy and key details of the ESMoL modeling language. We include a discussion of an interpreter development architecture to improve the extensibility and maintainability of the language and tools.

- Chapter IV gives an example of incremental syntactic analysis in ESMoL models.

- Chapter V covers incremental schedule calculation.

- Finally, chapter VI addresses lessons learned and potential future work in this area.
CHAPTER II

RELATED WORK

Modeling Languages and Tools for Embedded Systems Design

A number of projects seek to bring together tools and techniques which can automate different aspects of high-confidence distributed control system design and analysis:

- AADL is a textual language and standard for specifying deployments of control system designs in data networks\cite{14}. AADL projects also include integration with the Cheddar scheduling tool\cite{15}. Cheddar is an extensible analysis framework which includes a number of classic real-time scheduling algorithms\cite{16}.

- Giotto\cite{17} is a modeling language for time-triggered tasks running on a single processor. Giotto uses a simple greedy algorithm to compute schedules. The TDL (Timing Definition Language) is a successor to Giotto, and extends the language and tools with the notion of modules (software components)\cite{18}. One version of a TDL scheduler determines acceptable communication windows in the schedule for all modes, and attempts to assign bus messages to those windows\cite{19}.

- The Metropolis modeling framework\cite{20} aims to give designers tools to create verifiable system models. Metropolis integrates with SystemC, the SPIN model-checking tool, and other tools for schedule and timing analysis.

- Topcased\cite{21} is a large tool integration effort centering around UML software design languages and integration of formal tools.

- Several independent efforts have used the synchronous language Lustre as a model translation target (e.g. \cite{22} and \cite{23}) for deadlock and timing analysis.
• RTComposer[24] is a modeling, analysis, and runtime framework built on automata models. It aims to provide compositional construction of schedulers subject to requirements specifications. Requirements in RTComposer can be given as automata or temporal logic specifications.

• The DECOS toolchain [25] combines a number of existing modeling tools (e.g. the TTTech tools, SCADE from Esterel Technologies, and others) but the hardware platform modeling and analysis aspects are not covered.

We are creating a modeling language to experiment with design decoupling techniques, integration of heterogeneous tools, and rapid analysis and deployment. Many of the listed projects are too large to allow experimentation with the toolchain structure, and standardization does not favor experimentation with syntax or semantics. Due to its experimental nature some parts of our language and tool infrastructure change very frequently. As functionality expands we may seek integration with existing tools or standards as appropriate.

**Compositional and Incremental Methods**

In order to introduce the topic, we will first use some definitions from Edwards et al [26] to clarify terms and concepts in this research area. We have expanded the definitions and descriptions slightly to better fit our approach.

A *formal design* consists of the following elements[26]:

- A *specification* of system functions and behavior, including any details necessary to determine correctness with respect to requirements.

- A set of *properties* which the design must satisfy, that can be checked against the specification. These are derived from requirements or are assumed for correct operation of any system (e.g., deadlock-freedom).

- A set of *performance indices* allowing us to assess the quality of a particular design.

- *Constraints* on the performance indices. These are also derived from requirements.

Edwards et al further classify *properties* as follows[26]:
1. Properties *inherent* to the model of computation (behavior), which can be shown to hold for all specifications.

2. *Syntactic* properties can be determined by tractable analysis of the structure of elements in the specification.

3. *Semantic* properties can only be determined by examining the actual behavior of the specification. This means executing the specification either implicitly or explicitly over the full range of inputs.

In our work we consider proper design *specification* and correctness *properties*, but have not yet addressed *performance indices* and *constraints* beyond timing latency. In particular, we aim to create modeling tools and techniques which favor correct design by constructing model-based design environments that provide significant *inherent* properties for all well-formed models, or efficient analysis of *syntactic* properties. For *semantic* properties we seek abstractions which allow us to encode correct behavioral relationships into the syntax of the specification language, reducing expensive *semantic* analysis to less costly *syntactic* analysis.

We can now describe our approach to decoupling within this framework. We seek to achieve vertical decoupling by selecting a platform (TTA) which provides timing determinism and synchrony as *inherent* properties, and use passive control design methods to reduce the *semantic* property of robust stability to a *syntactic* concern. For horizontal decoupling we aim to use compositional techniques, which are inherently *syntactic* – for example, many properties in a design model can be evaluated from the bottom-up, following the design hierarchy. We also use incremental methods to make evaluation of both *syntactic* and *semantic* properties more compatible with iterative development processes.

**Incremental Scheduling Analysis**

Timing is a fundamentally semantic property. Determination of design validity and correctness depend on properties only verifiable by execution of the behaviors of the model. In a design model we can easily represent the relations between tasks, messages, and hardware for the purposes of synthesizing simulations or even scheduling analysis problem specifications. However, these structures
have little bearing on determining the actual admissibility of tasks and messages into an existing
design. We can always draw the connections in the model, but only semantic analysis will yield an
indication of whether or not the model is well-formed with respect to schedulability, and whether it
satisfies latency constraints.

In most cases, useful compositional and incremental techniques for scheduling must introduce
some restriction of behavior or approximation into a problem which is highly coupled over the entire
system design in order to reduce that coupling for scalable analyses. As we will see in the sequel,
for some scheduling algorithms and correctness criteria, compositional and incremental analyses
can proceed without introducing approximations into the behaviors represented by the design. In
these cases the properties are specified locally (i.e. task deadlines), greatly limiting the effects of
dependencies but also greatly limiting the ability to express and enforce constraints which meet end-
to-end deadlines. The much more difficult and general case considers end-to-end latency over the
dependency graphs between tasks and messages. This forces us to properly model global coupling
in the design, but seriously complicates our efforts to find useful decoupling methods.

**Hierarchical Schedulability Analysis Using Resource Interfaces**

Hierarchical schedulability analysis is a technique for abstracting a set of hard real-time components
in such a way that multiple task sets could efficiently be composed and analyzed. More specifically,
1) we can analyze heterogeneous schedulability models, where different groups of tasks are run
together under different scheduling algorithms; and 2) given a working, feasible real-time system, we
can efficiently determine whether new task sets can be admitted for execution, even if they run under
different scheduling algorithms. Admission depends on safety and resource availability. Beyond the
compositional and incremental structure of models for analysis, hierarchical scheduling also requires
runtime scheduling algorithms that support hierarchical resource sharing.

We deal with computing tasks and data communication messages whose respective execution
times and data transfer times are known and bounded for the contention-free case. In this section
we will refer to all resource consumers (tasks and messages) as tasks for simplicity. We consider only
periodic tasks (or sporadic tasks with a known maximum frequency). A resource provides a known
<table>
<thead>
<tr>
<th>Scheduling algorithm</th>
<th>Priority scheme</th>
<th>Compositionality</th>
</tr>
</thead>
<tbody>
<tr>
<td>Static schedule</td>
<td>Fixed, non-preemptive</td>
<td>Adding tasks incrementally requires a priori restrictions (such as harmonic periods), or recomputation of the whole schedule.</td>
</tr>
<tr>
<td>EDF (Earliest Deadline First)</td>
<td>Dynamic, preemptive based on the next nearest utilization, which can be calculated easily for the current workload.</td>
<td>Admitting new tasks is a function of utilization, which can be calculated easily for the current workload.</td>
</tr>
<tr>
<td>RM/DM (Rate or Deadline Monotonic)</td>
<td>Fixed, preemptive Priority determined by period or by relative deadline.</td>
<td>Admitting new tasks requires analysis against utilizations or demand bounds at all higher priority levels.</td>
</tr>
</tbody>
</table>

Table 1: Common real-time scheduling algorithms

amount of capacity over time, and tasks consume that capacity when executing. Schedulability implies that the resources supplied by scheduling algorithms are sufficient to meet the demand imposed by the tasks.

Easwaran [27] clarifies two fundamental approaches to compositional scheduling analysis:

1. A task set may be considered abstractly as a single task demand function under a scheduling algorithm which is global to all tasks. This approach was proposed by Wandeler [28]. Resources are abstracted under a supply bound function (sbf), and tasks are composed under a demand bound function (dbf) using the real-time calculus[29]. In this analysis approach the order in which tasks are analyzed can affect the satisfaction of the schedulability property for fixed-priority scheduling, so careful restrictions must be placed on the order of analysis.

2. A task set, its scheduling algorithm, and its resources can be seen as a new resource supply function, which is the approach we will cover here. This technique has the advantage that each component then presents a partial resource supply model to its child components, which may also be composite. The resulting structure is a hierarchy of tasks, each with its own resource supply function. During design and analysis, each set of supply-demand relationships is restricted to its own scope in the model hierarchy. The hierarchical scheduling approach was first proposed by Shin [1], and extended by Easwaran[27] to better model preemption overhead and deadlines.

A runtime scheduling algorithm controls the execution of a task set to ensure that all tasks get adequate resources. Scheduling algorithms are usually characterized by their priority policy. Table
### Table 2: Resource supply models and their parameters.

<table>
<thead>
<tr>
<th>Supply model</th>
<th>Parameters</th>
<th>Min sbf(t)</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bounded delay</td>
<td>interval ([t_1, t_2]) supply rate (c) delay bound (\delta)</td>
<td>(c(t - \delta)) if (t \geq \delta) 0 if (t &lt; \delta)</td>
<td>For any time interval ([t_1, t_2]), supply (c(t_2 - t_1)) units before time (t_2 + \delta).</td>
</tr>
<tr>
<td>Periodic</td>
<td>period (\Pi) supply (\Theta)</td>
<td>(t - (k + 1)(\Pi - \Theta)), if (t \in [(k + 1)\Pi - 2\Theta, (k + 1)\Pi - \Theta]) ((k - 1)\Theta), if not (k = \max) ((t - (\Pi - \Theta))/\Pi), 1)</td>
<td>Supply (\Theta) units every (\Pi) time units.</td>
</tr>
<tr>
<td>EDP (Explicit Deadline Periodic)</td>
<td>period (\Pi) supply (\Theta) deadline (\Delta)</td>
<td>(\left\lceil\frac{t-(\Delta-\Theta)}{\Pi}\right\rceil) (\Theta) + (\max{0,) (t - (\Pi + \Delta - 2\Theta) - \left\lceil\frac{t-(\Delta-\Theta)}{\Pi}\right\rceil) (\Pi})</td>
<td>Extends the periodic model with a deadline parameter (\Delta).</td>
</tr>
</tbody>
</table>

1 describes some common scheduling algorithms and their priority schemes, along with notes on the details of incrementally extending models under each particular scheduling algorithm.

For supply models a constant supply is most common (i.e., \(s(t) = c\)), but other models have better compositionality properties. Mok and Feng [30] introduced a bounded delay model for resource supply. Shin and Lee [31][1] presented a model where a resource is modeled to provide a fixed amount of supply at a constant periodic rate. Easwaran [27] extended the periodic supply model with support for user-specified deadlines. The models and their parameters are described briefly in Table 2.

The key concept is that we can specify a real-time component as a collection of periodic tasks, a scheduling algorithm, and a resource supply interface. The real-time component executes the tasks according to the specified algorithm, subject to the supply constraints provided by the resource model. This structure allows real-time components to be specified and executed as a hierarchy, as each component’s resource interface appears as a single periodic task to the component at the next higher level. The top level component provides the total (often constant) supply.

As an example we will describe the periodic supply model here in greater detail. A periodic resource \(\Gamma\) supplies \(\Theta\) units of resource every \(\Pi\) time units. The actual supply could occur anywhere in each time interval of length \(\Pi\), so we interpret the occurrence according to the worst case with respect to schedulability: supply first occurs as early as possible where it might be missed for a given analysis interval. In the worst case, the first supply is followed by a blackout interval of length \(2(\Pi - \Theta)\), followed by supply instances which occur as late as possible for all successive periods,
Figure 2: Worst-case analysis interval for determining the supply bound function of the periodic resource model $\Gamma$ for $k = 3$. Figure reproduced from [1, Fig. 4.1].

$$sbf_{\Gamma}(t) = \begin{cases} 
  t - (k + 1)(\Pi - \Theta) & \text{if } t \in [(k + 1)\Pi - 2\Theta, (k + 1)\Pi - \Theta] \\
  (k - 1)\Theta & \text{otherwise}
\end{cases}$$

$$k = max \left(\left\lceil \frac{t - (\Pi - \Theta)}{\Pi} \right\rceil, 1 \right)$$

(1)

as shown in Fig. 2. In the figure $t$ is the length of the analysis interval, starting at the point of worst-case supply. The interval $t$ starts with the blackout period $2(\Pi - \Theta)$, marked by the dark dashed arrows. During the blackout period the interface provides no supply. This figure shows an analysis interval with three periods, and ending in the middle of a capacity interval.

Eq. 1 shows an expression for a supply bound function representing the worst-case supply for a periodic resource interface as depicted in Fig. 2. For more tractable analysis, a linear supply function is commonly used, as in Eq. 2.

Finally, we give the schedulability condition for fixed-priority systems as described in [1, Theorem 4.5.3]:

“A scheduling unit $SU(W, R, A)$ where $W$ is a periodic [task] workload set, $R$ is a periodic resource $\Gamma(\Pi, \Theta)$, and $A$ is the RM scheduling algorithm, is schedulable (with worst-case resource supply of $R$) iff

$$\forall T_i \in W, \exists t_i \in [0, p_i] \text{ dbf}_{RM}(W, t_i, i) \leq sbf_{\Gamma}(t_i).$$

”

$$sbf_{\Gamma}(t) = \begin{cases} 
  \frac{\Pi}{\Theta}(t - 2(\Theta - \Pi)) & \text{if } t \geq 2(\Pi - \Theta) \\
  0 & \text{otherwise}
\end{cases}$$

(2)
The final relevant result we will discuss creates an periodic resource interface to model the supply required by child tasks in a hierarchical scheduling model. This technique is only valid for EDF scheduling, and includes preemption overhead.

[27, Def. 4.4] **Multi-period composition** For the child component $C_i$, let $k_i$ be the period at which its interface provides resources. $\phi_{k_i} = (k_i, \Theta_{k_i})$ is the resource model in the interface $I_{C_i}$. For parent component $C$ and its operating period $k$, $\Theta_k$ can be found using the optimization problem in Eq. 3.

$$\text{minimize } \Theta_k$$
$$\text{subject to } \frac{\Theta_k}{k} \geq \sum_{i=1}^{n} \Theta_{k_i} + PO(k_i)$$
$$2(k - \Theta_k) \leq \min_{i=1...n} \{\frac{2(k_i - \Theta_{k_i} - k_i PO(k))}{2}\}$$

Here $PO(k)$ is the preemption overhead function, which bounds the demand contributed by system overhead for a given interface period $k$. $PO(k_i)$ is then the overhead contributed by the interface of component $C_i$ due to the period mismatch between the resource interfaces for $I_{C_i}$ and $I_C$. $PO$ is given as a fraction of the period $k_i$. One suggested form for the preemption overhead function is $PO(k) = \frac{A}{k}$, where the constant $A$ is specified by the designer from calibration data. Multi-period composition for fixed-priority scheduling models and periodic resource interfaces is still an open problem.

For dynamic scheduling, deadlines and dependencies complicate compositionality when considering end-to-end properties in a dataflow network. Adjusting the deadline of a single task can lead to a loss of schedulability (or performance) for components in other parts of the system, affecting tasks which may not have direct functional dependencies. Dependencies are implemented using offsets or synchronization mechanisms in dynamic scheduling environments. Offset determination essentially computes a static schedule for a subset of the tasks in the system to ensure they meet end-to-end latency requirements. For explicit task synchronization, unbounded nesting of locks across different priority levels can lead to undecidable response times for scheduling models. Platforms which provide priority inheritance protocols can alleviate many situations arising from such task nesting.
situations. The priority ceiling protocol explicitly limits the depth of chains of blocking calls between processes at different priority levels, so response time bounds can always be calculated[32].

**Incremental Techniques for System Designs**

Static scheduling is generally not compositional. Kwok and Ahmad give a detailed survey and evaluation of static task graph scheduling methods, including a discussion of the abstractions on which those methods are based[33]. Schild and Würtz [34] and Ekelin and Jonsson [35] describe constraint models for static real-time scheduling problems, such as would be used in designs based on time-triggered communication systems. Zheng and Chong give an alternate formulation of the constraint models which is more complex, but allows the designer to optimize for extensibility[36]. The additional complexity is due to the addition of constraints and variables to model preemption as well as the need to explicitly model task and message finish times in order to model slack. Using preallocated slack to achieve incrementality is only a partial solution, as the technique only addresses the availability of time to modify or change tasks and messages. It is a necessary condition, but not sufficient. In order to have a sufficient condition we would also have to address changes or additions to the data dependency graph.

For statically scheduled workloads Zheng and Chong give the following form (Eq. 4) for their slack metrics[36]. Maximizing slack creates additional time in the schedule for modifying or adding tasks and messages.

\[
M_E = \sum_{(t_i, t_j) \in \bar{\omega}} w_{t_i, t_j} \times ( (s_{t_i, t_j}^{m} - f_{t_i}) + (s_{t_j} - f_{t_i, t_j}^{m}) )
\]  

In Eq. 4 the set \( \bar{\omega} \) is the set of task instance pairs which have remote data dependencies. \( t_i \) is task instance \( i \), \( s_{t_i} \) is a variable representing its start time, and \( f_{t_i} \) is its end time. \( s_{t_i, t_j}^{m} \) is the start time of the message from task \( t_i \) to task \( t_j \), and \( f_{t_i, t_j}^{m} \) is the corresponding end time. \( w_{t_i, t_j} \) is a designer-specifiable optimization weight for each task pair in \( \bar{\omega} \).

In Pop et al[37] the authors describe an incremental approach to the allocation, mapping, and scheduling problems for a time-triggered platform with non-preemptive statically scheduled tasks and TDMA communications. The task graph granularity of their models is very coarse, where a
vertex corresponds to an application, and edges correspond to dependencies from the perspective of software maintenance. Two application vertices are dependent if modifying the first will also require the modification of the second. Their approach relies on the specification of size and probability parameters to control the provision and distribution of schedule slack in order to accommodate future capacity. They assume a discrete set of possible task configurations, and then the designer gives probability values to each element, based on the likelihood of needing to add something similar to a future design. Their quality metrics are based on a cost function which combines slack size and distribution. Bin packing is used to maximize the available slack, and a simulated annealing approach is used to maximize the objective function.

Matic proposes an interface algebra which deals compositionally and incrementally with delay and data dependencies given task arrival rates, latency bounds, task dependency graphs, and WCET bounds. The algebraic formalism describes composition, interconnection, abstraction, and refinement of components. Within these operations, the model structures jointly evaluate schedulability (using the bounded-delay resource supply model of Mok and Feng[30]), causality, and end-to-end delay[38]. The end-to-end delay bounds are specified cumulatively rather than globally, and the author does not consider the conservatism of the formalism with respect to the behaviors that it represents.

Ghosh et al present a formal model for allocation (Q-RAM) which searches over possible quality levels (bandwidths), delay levels (hops), and routes for a set of communicating tasks[39]. The immense size of the search space is pruned by considering the hierarchical structure of the network, and exploiting locality of communication where possible. The objective is to flexibly, scalably, and incrementally determine resource allocation on the network while maintaining near-optimality for a utility metric. In [40] the authors present a distributed allocation scheme where each sub-domain negotiates for a common global set point for its allocated tasks. Their approach is fundamentally suited to incremental analysis, though their evaluations do not stress this aspect.
Incremental Deadlock Analysis

We consider compositional and incremental techniques related to deadlock detection or avoidance in embedded system designs specified as dataflows. For many of these models deadlock-freedom is an inherent property. Functional determinism is another important property inherent in dataflow models of computation, but which we will not cover in detail. We shall rely on the fact that synchronous data flows (SDF) exhibit both properties inherently. First, it will be important to review some of the historical and current work in deadlock analysis.

Overview of Compositional Methods

Early work in semantics for distributed computing languages considered functional semantics – for example, determining whether a given distributed dataflow network would deterministically calculate a specified function [41]. Kahn showed that under proper assumptions, the network could calculate the same function deterministically regardless of the firing order of the network. Kahn’s approach used Scott continuity[42] to ensure the existence of a unique fixed point for the dataflow network. The difficulty with this formalism is that while it provides compositional deadlock freedom, scheduling a Kahn network is not compositional – although the data flow elements and their interconnections are specified independently, scheduling or other analysis of the network may require global determination of a fixed point (via iteration or symbolic analysis) in order to yield a result. Maximum capacities of data buffers between components are also not decidable in the original Kahn formalism. Synchronous data flow models of computation are a subset of the Kahn formalism for which the firing orders and maximum buffer sizes can be precalculated [8]. These data flow formalisms are compositional, and if structured correctly can be used for incremental analysis.

SDF actors (functional components) are constrained to have fixed data token consumption and production rates at each clock tick (firing). Balance equations on the topology of the data flow can indicate whether the specified token flows are consistent, or whether each actor can be assigned a specific number of firings to satisfy the rates in the specification [8]. For flow rate specifications, Buck discusses the consequences of allowing richer data token flow models in the specification, such as conditional execution and nondeterminism[9]. These constructs can easily lead to undecidability for
token flow rates on various data links and therefore lead to undecidability for maximum buffer sizes.

Lee and Parks illustrate that balance equations compose easily in hierarchical SDF specifications[43]. Balance equations may be given independently for subcomponents and solved to abstract the token flow within the parent component to appear as a single actor having fixed token production and consumption rates. For flow rates adding an actor or changing an existing actor may require firing order sequence recomputation from the containing component to the top of the model hierarchy, so the effects of incremental design changes on the analysis could be isolated.

As an example of the formalism, static scheduling of a SDF subsystem on a particular platform means determining the firing order of all of the components, including the number of times each component is fired. These values are determined using balance equations as described by Lee and Messerschmitt[8]. In [8, Eq. 3] Lee gives a dynamic equation relating the amount of data in each of the buffers to the firing sequence of the nodes and the topology of the SDF graph. We repeat some of the discussion here to illustrate another fundamental model form for SDF analysis:

Let $b[k] \in \mathbb{Z}^m$ represent the quantity of data in each of the buffers at discrete time tick $k$. Here $m$ is the number of edges in the SDF graph, associating one vector component with each buffer. Let $v[k] \in \{0, 1\}^n$ represent whether each node is fired at time $k$, where the components of the vector $v$ range over the nodes. Let the matrix $\Gamma$ be defined as follows for the SDF graph:

$$
\Gamma_{i,j} = \text{token rate}(\text{node}_j, \text{edge}_i)
$$

$\text{node}_j$ is the $j^{th}$ node (component) in the graph, $\text{edge}_i$ is the $i^{th}$ edge (as in buffer vector $b$, above), and $\text{token rate}$ is the number of tokens produced on $\text{edge}_i$ by a single firing of $\text{node}_j$. For nodes consuming data the value is negative. Then [8, Eq. 3] is given as

$$
b[k + 1] = b[k] + \Gamma v[k].
$$

From the topology matrix $\Gamma$ we can solve for the number of firings required to balance the SDF graph or subsystem. If $\Gamma$ has rank $n - 1$, then a positive integer vector $q$ exists such that $\Gamma q = 0$. $q$ represents the firing quantities. This is proved by Lee in [44].
More recently, Gossler and Sifakis proposed a formalism for modeling and analyzing asynchronous designs based on a specification language known as BIP (for Behaviors, Interactions, and Priorities)[45][46]. Component behaviors are modeled as automata, interactions as structures on the sets of possible event combinations between connected components, and priorities are global restrictions of the possible interactions. Bliudze and Sifakis give a formal definition for (possibly asynchronous) event interactions represented by connectors in BIP[47]. These connectors can be nested, and algebraic techniques are given for reducing hierarchical connectors to simpler representations. This provides the foundation for incremental design, as an existing (reduced) algebraic model for the connectors in a design provides an interface for extending the design with additional components and their interactions. New interactions can be “connected” to the existing connector structure and analysis performed with respect to the reduced interaction model for those connectors. Bensalem et al [48] give a detailed description of a formal model for incrementally constructing the interaction space and efficiently computing behavior invariants for deadlock verification. These techniques take a step towards reducing the semantic analysis required for deadlock analysis to a syntactic analysis problem.

Ferrarini[49] deals with compositional design by giving the designer a safe set of building blocks which allow the incremental construction of discrete control systems which satisfy boundedness, cyclicness, and liveness. The analysis is reduced to a graph based on connections among tasks.

**Synchronous Distributed Platforms**

In standard real-time systems, distributed platforms typically do not provide fully synchronous semantics. Synchronization between processes is provided by explicitly specified locking primitives such as semaphores, mutexes, and monitors. Cyclic dependencies in these specifications can lead to deadlock if they are not sequenced correctly. In addition, scheduling of tasks at different priorities along with their dependencies can lead to deadlock or starvation (effective deadlock). For dynamically scheduled tasks, the priority ceiling protocol can completely avoid priority-related deadlocks by bounding the depth of chains of processes waiting on one another and by implicitly ordering the acquisition and release of locks[32].
The *Time-Triggered Architecture* (TTA) relies on *a priori* knowledge of message schedules to achieve and maintain synchronous execution over distributed processors. Each processor has a full copy of the message schedule, and all messages are sent and received at precise, precalculated times. Schedule-driven execution eliminates the need to send control and acknowledgment signals between processing nodes, reducing horizontal coupling in the design.

The basic semantic assumptions of the TTA can be summarized as follows:

1. All clocks in the system are synchronized, and the schedules run over a global periodic cycle.
2. Data transfers occur on a common, shared bus where all messages are broadcast to all processors.
3. Data reading and writing are non-blocking for tasks.
4. Data message updates occur outside the execution window for sending/receiving tasks.
5. Messages on the bus do not preempt each other.

The Timed-Triggered Protocol (TTP) realizes the communication mechanism within the TTA. Each processing node receives a TDMA time slot in which to send messages. TTP provides the following services (see [50] for details):

- Timed message transport between nodes, reducing latency and jitter for individual transfers.
- Fault-tolerant distributed clock synchronization on all nodes, ensuring deterministic mode change behavior for replicated computations.
- Fault-tolerant membership service for all nodes.
- Clique avoidance when faulty nodes are isolated from the network.

The *Loosely Time-Triggered Architecture* (LTTA) attempts to maintain the synchronous capabilities of the TTA without the full clock synchronization between nodes, in order to reduce coupling in the hardware architecture. LTTA is based on the following assumptions:

1. Assumption 1
• Each processing node has an independent local clock.

• Each shared variable (message) has a separate communication channel.

• Data reading and writing is performed independently at all processors without synchronization.

2. Assumption 2. Each update (write) is broadcast to all nodes.

3. Assumption 3. Each cycle in the data flow graph has at least one unit delay (no causality loops).

4. Assumption 4. Each communication between processors is delayed by at least one tick.

Additional assumptions relate to particular implementations of LTTA (see Benveniste[51] for details). Causality loop conditions can be problematic to analyze. Often in a dataflow graph many paths and loops are interconnected, complicating the analytic enumeration of loops to ensure adequate buffer placement and cycle initialization. The simplest solution is structural – to buffer all data transfers, as in Kahn networks[41]. Zhou and Lee discuss some of the difficulties in performing loop analysis for cycles in synchronous dataflows[52].

Tripakis et al describe a synchrony-preserving map from concurrent data-driven synchronous Mealy automata to an LTTA platform which relies on back pressure and skipping to avoid deadlocks[53]. In their formalism, buffer sizes are fully decidable and the authors give a lower bound for the maximum required buffer size to prevent deadlock. The authors use an argument based on the Kahn process network formalism in order to guarantee determinism in their model, which allows data-driven mode switching in components[53].

**Incremental Causality Profiles**

These techniques seek for mathematical structures which ensure deadlock-freedom as a syntactic property of dataflow models. Zhou and Lee describe an algebraic model which abstracts data dependencies between interconnected actor ports in order to determine liveness. It is based on repeated reductions on algebraic expressions representing connectivity[52].

21
Tripakis et al describe a method for addressing the cycle token constraint compositionally in hierarchical SDF models [2]. They propose the creation of a subsystem profile which abstracts the dependency information inside the subsystem and provides interface FIFOs where necessary in order to break causality loops. The causality profile technique seems to provide the right level of support for incremental component addition and for change isolation with respect to the causal dependencies within the component and in the larger design. The one drawback is that the profile fundamentally changes the component from a pure SDF model to a synchronous block with shared FIFOs on its ports. As an example we will consider Tripakis’ approach in greater detail.

In order to motivate the approach, consider the effects of an addition or a change to a flat SDF model. Adding a new actor to the model would require a reformulation of the balance equations, followed by a deadlock assessment by simulating token flows as described in Lee[8]. A similar analysis would follow for changes to an existing design, if those changes affected the token flow rates or connectivity of existing components. Adding hierarchy to SDF graphs helps encapsulate token flow rate calculations. Each subcomponent can be abstracted as a single component with fixed token flow quantities for its I/O ports. Deadlock assessment is more problematic for complex hierarchical designs. The number of operations for token flow simulations to assess deadlock in an SDF graph are bounded by the total number of actor firings in the system, as determined by the minimal solution to the balance equations. The number could be large. For example, a composite subsystem could fire $N$ times in the simulation, and have subsystems each of which fire $N$ times to a hierarchy depth of $M$, for a total of $O(N^M)$ actor firings. Such an expensive analysis prohibits incremental design and does not isolate changes well.

As with incremental scheduling, we would like to have an interface for each component which represents the behavior of the component with respect to deadlock analysis. Tripakis et al provide such an interface called a DSSF profile (Deterministic SDF with Shared FIFOs), with care to reduce the conservatism of the interface abstraction in order to keep the technique useful. An added benefit of these profiles is that they use the knowledge of the order of actions in the firing interface to safely share input and output FIFOs between ports, reducing the buffer space required for implementation[2].
A DSSF profile is simply a multigraph which has four types of nodes: firing function nodes, input signal nodes, output signal nodes, and external FIFO nodes. Edges represent dataflow dependencies. Firing function nodes represent the firing of the actor when the order of actions is determined. Edges into or out of a firing node are labeled with the quantity of data tokens consumed or produced by that node when firing. For simple monolithic actors, the DSSF profile is simply a single firing node for the actor function with input and output edges for its parameters. The edges are labeled according to their token flow rates. Constructing a DSSF profile for composite actors is more involved. The steps are outlined here, from Tripakis [2, Section 7].

1. To create a DSSF profile of a composite actor $P$, first connect the DSSF profiles of its subcomponents according to the topology of the dataflow graph of $P$.

2. Using the newly created DSSF profile graph of $P$, solve the balance equations to determine the required number of firings for each subcomponent (i.e., find the repetition vector for the nodes as described above). If the balance equations are inconsistent, then the specification for $P$ is also inconsistent.

3. Simulate the DSSF profile graph as an SDF graph to analyze for deadlock. Initialize each cycle in the DSSF graph with a data token. If the firing nodes in the graph can each be fired the proper number of times, then no deadlock exists.

4. Unfold the analyzed DSSF graph to create a directed acyclic graph (DAG) that represents the I/O dependencies of $P$. There are two steps:

   (a) First, replicate each firing node in the graph according to its number in the repetition vector. Replicate associated input and output nodes. Add dependencies between the replicated nodes so that each set of repeated nodes is sequentially ordered. Also replicate the edges into and out of interface FIFO nodes as necessary (without replicating FIFO nodes).

   (b) Replace the internal FIFO queues with explicit dependencies, according to the formula given in Eq. 5. Let $A_1, A_2, \ldots, A_a$ be the set of firing functions producing data for the FIFO, and let $B_1, B_2, \ldots, B_b$ be the set of firing functions consuming data from
the FIFO. These firing functions come from the DSSF profile graph before replicating the instances. Now consider a particular producer and consumer pair, $A_v$ and $B_u$, where $v \in [1 \ldots a]$ and $u \in [1 \ldots b]$. In the instance-replicated graph, the $i_{th}$ instance of $A_v$ can be written $A_{v,i}$ and the $j_{th}$ instance of $B_u$ can be written $B_{u,j}$. Then for all combinations of producer/consumer pairs $(v, u) \in [1 \ldots a] \times [1 \ldots b]$ and for each particular $(v, u)$ consider all of the instances $(i, j) \in [1 \ldots r_{A_v}] \times [1 \ldots r_{B_u}]$. Then create a dependency directly between $A_{v,i}$ and $B_{u,j}$ if the following condition is satisfied:

$$
\begin{align*}
    d + (i - 1) \sum_{h=1}^{a} k_h + \sum_{h=1}^{v-1} k_h \\
    < (j - 1) \sum_{h=1}^{b} n_h + \sum_{h=1}^{u} n_h
\end{align*}
$$

(5)

$d$ represents the number of initial tokens in the FIFO. $k_h$ is the number of tokens produced in the FIFO by actor $A_h$. $n_h$ is the number of tokens consumed from the FIFO by actor $B_h$. If the link between an producing actor $A$ and a consuming actor $B$ is direct (i.e., no explicit FIFO node exists between them), then repeat this procedure with $a = b = 1$ to create the appropriate links between the instances.

The unfolded graph is used in the next step to get clusters that represent the firing functions of a new DSSF profile for the whole component. Having separate firing functions means that the abstracted component interface can be safely analyzed for deadlock without approximating.

5. Cluster the unfolded graph. The clusters are created so that no cyclic dependencies exist among clusters. The DAG clustering algorithm given in [2] aims for maximum reusability, where no false input-output dependencies will be created, and where the optimal (maximum) number of clusters or more will be created using a greedy technique (since the optimal technique is NP-complete). Clusters are also pairwise disjoint. Their algorithm is called greedy backward disjoint clustering. Backward clustering means that the algorithm works from the outputs backward. The result is a set of clusters $C_1, C_2, \ldots C_q$ which represent 'independent' subsets of dependencies for the component with respect to deadlock analysis.
6. Finally we create the new DSSF profile for the composite actor $P$. This step is somewhat involved. Let $P_c$ be the initial connected profile (from Step 1), let $D$ be the unfolded graph, let $C = \{C_i\}$ be the set of clusters from the previous step, and let $P_f$ be the new abstract profile graph that we’re trying to create.

(a) In $P_f$, create an atomic firing node $P.f_i$ for each cluster $C_i$ in $C$.

(b) For each input and output port of $P_c$, create a single FIFO. Each cluster writes to or reads from the FIFOs at a rate specified by the sum of the actors in each cluster connected to the FIFO. This can be determined from $P_c$, $D$, and the clusters $C_i$. If the rate is zero, then the cluster is not connected to the corresponding FIFO.

(c) Create dependency edges in $P_f$ between the firing nodes $P.f_i$ and $P.f_j$ if iff there exists at least one edge between any node of $C_i$ and any node of $C_j$.

(d) Consider each FIFO $L$ in $P_c$. Let $W_L$ and $R_L$ be the ordered collection of clusters, respectively, that write to and read from $L$. Create one dependency edge in each of $W_L$ and $R_L$ from the last cluster to the first, with an initial token on the edge. This encodes the requirement to wait to re-fire the cluster set until the last round has finished. If there is only one cluster in either $W_L$ or $R_L$, then do not create the additional edge.

(e) Now consider only the internal FIFOs of $P_c$ (i.e., those not connected to an external port). Suppose that a given internal FIFO $L$ has $d$ initial tokens. Let $m$ be the number of tokens produced by the clusters writing to $L$. $m$ should also be the number of tokens consumed by the reading clusters (by construction). Now we build a graph between the producer and consumer clusters as follows: Let the ordered set $z_0, z_1, \ldots, z_{m-1}$ represent the tokens produced by the clusters on the consumer side, ordered and partitioned according to the cluster order and the number of tokens each cluster consumes. Likewise, create the ordered set $w_0, w_1, \ldots, w_{m-1}$ for the producing clusters, similarly ordered and partitioned. For $i = 0 \ldots m - 1$, connect the cluster with token $z_i$ to the cluster with token $w_j$, where $j = (d + i) \mod m$. Also place $\lfloor \frac{d + m - 1 - i}{m} \rfloor$ initial tokens for the edge associated with token $w_i$. 

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(a) Composite actor example for DSSF (b) Unfolded graph. The dashed outlines generation. Actors A and B are the com-ponents of composite actor H. Each input beled with its cluster number (Ci) and output port is labeled with the token flow quantity. The center edge has six initial tokens.

(c) Final DSSF profile. Unlike the graph corresponding to the original diagram, this representation does not introduce false dependencies.

(f) Clean up the edges created in the last step as follows: remove all self-loops without initial tokens, and then combine multiple edges between clusters by summing the initial tokens and token flow values.

We take a simple example from [2, Fig. 10] to illustrate some of the steps in the technique. Fig. II.3(a) shows a composite actor specification for which we would like to create a profile. After the connection, unfolding, and clustering steps (1-N), we end up with the graph in Fig. II.3(b). This is used to create the final profile (Fig. II.3(c)), which abstracts both token flow rate and deadlock characteristics of the original composite actor H.
Compositional Stability Analysis

Compositional techniques in control systems analysis seek to verify stability of a design model using component properties and interconnection rules. Robust control techniques consider feedback interconnections of components, seeking to characterize and formalize behavior properties of the connected components when one or both of the components are perturbed. Teel briefly describes the development of the robust control approach for stability [54, Section IV]. The robust approach centers on the explicit modeling of uncertainty in system inputs and parameters, along with techniques for analyzing and optimizing interconnected systems designs that include uncertainties[55].

Passivity is a property of control system behavior that implies stability, composes under particular interconnections of components, and which reduces the destabilizing effects of data and parameter quantization [10] as well as delays due to digital processor scheduling and network communications [56] [57].

Passive Control Design

Let $E_{in}, E_{out}$ represent energy input and output for a component. Passivity means that energy output for a particular component never exceeds its energy input together with any remaining stored energy. Passive systems are compositional, as discussed below. Bounded-input bounded-output (BIBO) stability requires $E_{out} \leq KE_{in}$. BIBO stability follows directly from passivity, but is very weak in terms of our ability to direct the trajectory of the controlled system. Asymptotic stability is a stronger form of stability that implies convergence of the system trajectories to a particular point. If the system can be structured to provide a reference trajectory, then asymptotic stability permits trajectory tracking. Usually a search for a Lyapunov function is used to establish asymptotic stability for an arbitrary nonlinear system. Lyapunov functions generally represent (or bound) the behavior of the entire system, so they are not compositional. Passivity can be used to compositionally establish asymptotic stability with a few more strictness constraints.

To establish passivity for linear system models, Kottenstette gives Linear Matrix Inequality(LMI) conditions to verify passivity of a linear time-invariant system in state-space form[58] (Eq. 6). If a positive definite solution $P_i$ exists satisfying the given matrix inequality, then the component is...
passive. The inequality is interpreted in the sense of semidefinite programming, where for example $M \leq 0$ means “matrix $M$ is negative semidefinite”[59]. Further constraining the matrix $P$ to be symmetric, we can solve this LMI efficiently using convex optimization techniques[60].

$$\begin{bmatrix} A_i^T P_i + P_i A_i & P_i B_i - \frac{1}{2} C_i^T \\ B_i^T P_i - \frac{1}{2} C_i & -\frac{1}{2} (D_i^T C_i + C_i D_i) \end{bmatrix} \leq 0$$ \tag{6}

Desoer and Vidyasagar give frequency domain conditions for passivity for LTI system models[61].

For continuous-time:

$H$ is passive iff $H(j\omega) + H^*(j\omega) \geq 0$, $\forall \omega \in \mathbb{R}$.

For discrete-time:

$H$ is passive iff $H(e^{j\theta}) + H^*(e^{j\theta}) \geq 0$, $\forall \theta \in [0, \pi]$.

**Sector Analysis**

Using the formulation in Zames[62], the sector bounds for a possibly nonlinear control component are a real-valued interval $[a, b]$, where the endpoints come from the expression in Eq. 7.

$$\|y_T\|^2 - (a + b)\langle y, u_T \rangle + ab\|u_T\|^2 \leq 0 \tag{7}$$

For linear (and some nonlinear) system models, sector bounds may be computed symbolically during system analysis. Each component is assigned a real interval $([a, b] - \infty < a \leq b \leq \infty, b \geq 0)$ representing a range of possible input/output behaviors. Components whose bounds fall in the interval $[0, \infty]$ are passive (and have some notion of stability). Zames also presents rules for computing sector bounds for systems based on calculated component bounds and different types of interconnections between components [62]. We describe the sector formulas, rules, and bounds in greater detail below.

From [62] and [63], another way to look at Eq. 7 is the following formulation. A system $H$, ($y = Hx$) is inside the sector $[a, b]$ if $a \leq b$ and

$$\langle (Hx)_t - ax_t, (Hx)_t - bx_t \rangle \leq 0 \tag{8}$$
Where $\langle \cdots, \cdots \rangle$ is the inner product on the appropriate function space.

Most approaches to nonlinear control system design rely on continuous time assumptions. When we consider discrete time implementation in software subject to network delays and finite-precision quantization effects, linear approximations and high sample rates are used to obtain tractable analysis and realizable execution. In practice we have found that compositional techniques based on passivity have allowed us to construct reasonably low data rate digital controllers for nonlinear systems without resorting to conservative linear approximations\[64].

Passive control techniques have proven successful for many cases of nonlinear continuous time controllers, but nonlinear discrete time control poses several challenges. Unfortunately many control structures are not passive in discrete time. If we can approximate our controlled system as a cascade of passive systems then we can apply a systematic control design strategy, for which stability can be validated online.

Digital control for nonlinear physical systems with fast dynamics (such as a quadrotor helicopter) use a zero-order hold to convert control values produced at discrete time instants into step functions held over a continuous interval of time. For certain inputs and state trajectories, the hold process can introduce small amounts of new energy into the environment, violating passivity. The sector bounds analysis proposed by Zames \[62\] can be used to assess the amount of “active” (energy-producing) behavior which we can expect from a design under nominal operating conditions.

Zames’ critical insight was that many causal nonlinear systems’ dynamic input-output relationships can be confined to being either inside or outside a conic region. Systems whose input-output relationships can be confined inside a conic region are known as interior conic systems. Equivalently, these interior conic systems can be described as residing inside the sector $[a, b]$ in which $a$ and $b$ are real coefficients\[62\]. If there exist a real coefficients $a$ and $b$ such that Eq. 7 is satisfied then the system is an interior conic system inside the sector $[a, b]$ conversely if the inequality of Eq. 7 is reversed the system is exterior conic and outside the sector $[a, b]$. Table 3 describes the quantities used in Eq. 7. For linear time invariant (LTI) single input single output (SISO) systems the term $a$ is the most negative real part of its corresponding Nyquist plot, it therefore is an approximate measure of the phase shift of a stable system. A passive system is equivalent to an interior conic
Figure 4: Block diagram interconnection examples for conic system composition rules.
<table>
<thead>
<tr>
<th>Quantity</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u(t)$</td>
<td>Input signal</td>
</tr>
<tr>
<td>$y(t)$</td>
<td>Output signal</td>
</tr>
<tr>
<td>$|y_T|^2_2$</td>
<td>Energy produced by the component so far (output) in a time interval of length $T$.</td>
</tr>
<tr>
<td>$|u_T|^2_2$</td>
<td>Energy received by the component so far (input) in a time interval of length $T$.</td>
</tr>
<tr>
<td>$\langle y, u \rangle_T$</td>
<td>Correlation between the input and output sample values in a time interval of length $T$. This is a measure of dissipation.</td>
</tr>
<tr>
<td>$a$</td>
<td>Real-valued lower bound for the sector.</td>
</tr>
<tr>
<td>$b$</td>
<td>Real-valued upper bound for the sector.</td>
</tr>
</tbody>
</table>

Table 3: Quantities for the sector formula.

system which is inside the sector $[0, \infty]$ therefore a passive LTI SISO system has no more than $+/ -90$ degrees of phase shift in which all real parts of its corresponding Nyquist plot are positive real.

A conic system can also be modeled as a functional relation between the possible input and output signal spaces. This corresponds intuitively to a causal block diagram where the function specified in the block relates the inputs to the outputs, as in Fig. 4 (a). See Zames for a complete formal functional description of sector analysis. Given conic relations $H, H_1$ with $H$ in $[a, b]$ and $H_1$ in $[a_1, b_1]$ ($b, b_1 > 0$), and given a constant $k \geq 0$, we have the following sector composition rules from [62]:

1. $I$ is in $[1, 1]$ (Fig. 4 (b))
2. $kH$ is in $[ka, kb]$ (Fig. 4 (c))
3. $-H$ is in $[-b, -a]$ (Fig. 4 (d))
4. sum rule $H + H_1$ is in $[a + a_1, b + b_1]$ (Fig. 4 (e))
5. inverse rule(s) (Fig. 4 (f))

   (a) $a > 0 \rightarrow H^{-1}$ is in $[\frac{1}{b}, \frac{1}{a}]$.

   (b) $a < 0 \rightarrow H^{-1}$ is outside $[\frac{1}{a}, \frac{1}{b}]$.

   (c) $a = 0 \rightarrow (H^{-1} - (\frac{1}{b}I))$ is positive.

For rule 5 an inverse system model must be well-defined (i.e. exist), as in the case of invertible linear system models.
The sector composition rules illustrate the compositional nature of sector analysis. Zames also gives conditions under which feedback-interconnected conic systems exhibit stability [62, Theorems 2a, 2b]. We will not describe all of the details here, but simply relate the sufficient condition described in Kottenstette to establish stability of feedback control loops where the included systems are conic [65, Corollary 2]:

Assume that the combined dynamic system $H : [u_1, u_2] \rightarrow [y_1, y_2]$ depicted in Fig. 5 consists of two dynamic systems $H_1 : u_1 \rightarrow y_1$ and $H_2 : u_2 \rightarrow y_2$ which are respectively inside the sector $[a_1, b_1]$ and strictly inside the sector $[0, 1 + \epsilon]$, for all $\epsilon > 0$. Then $H$ is bounded ($L^m_2$ stable for the continuous time case or $l^m_2$ stable for the discrete time case) if:

$$\frac{-1}{\max\{|a|, b\}} < k < \frac{-1}{a_1}, \text{ if } a_1 < 0$$

$$-\frac{1}{b} < k < \infty, \text{ otherwise.}$$

In order to design with sectors, it is important to be able to relate sector conditions for continuous-time systems with those for discrete-time systems. Kottenstette et al. introduce a linear discretization operator called the IPESH-transform (Inner Product Enhanced Sample and Hold) that preserves sector conditions during discretization [12]. Briefly put, the IPESH-transform guarantees that a continuous-time conic system $H_{ct}$ that is inside a sector $[a, b]$ that is discretized using IPESH at a sample rate of $T_s$, then the resulting filter will lie in the sector $[aT_s, bT_s]$ [12, Lemma 3]. Finally, we give the form for the transformation (Eq. 9).
Sector analysis extends our analysis capabilities to many nonlinear and non-passive system models and control structures. As discussed in the previous section, the mathematical framework by itself is not sufficient to support incremental design analysis. We must deal with partitions of the design into interfaces and the relationships of various control loops to each other and to the plant dynamics. This is an open problem in modeling, and sector analysis is one candidate for a formal analysis framework which could support incremental design and analysis tools.

**Passivity and Platform Effects**

Modern passive control techniques demonstrate that passive systems exhibit insensitivity to platform effects. We will briefly cover a few of these results.

1. **Quantization errors** Perhaps the best passivity-based framework for quantization effects is described by Fettweis [10]. Physically realizable passive circuit models are used to derive compositional digital filter structures with similar passivity constraints.

2. **Sampling rate variations** (IPESH/multi-rate) Kottenstette presents a technique for discretizing filters which is less sensitive to changes in sampling rate (IPESH transform) [11]. He also presents a method for constructing passive networked control designs where individual components operate at different rates [12].

3. **Network delays** For communication delays, Anderson and Spong [66] derived passivity conditions for a particular networked control configuration. The two-port scattering matrix formulation make the delay analysis tractable, as it appears as a two-port component in the center of the control structure. From this they derive passivity conditions. Their digital implementation was unstable for large time delays. The control analysis used continuous-time models, and a naive application of discretization.

Niemeyer and Slotine extended the work of Anderson using the observation that physical waves transmit energy in a stable manner [13]. They introduced discrete wave variables, which
surprisingly removed the delay term from the original formulation. This led to a much more
general framework for handling communications in passive control designs. Wave variables
redistribute the values of the effort and flow at a port consistent with the distributed effort
and flow that a wave would use to transmit energy between two points.
Consider the general class of control system designs for use in a flight control system. Sensors, actuators, and data networks are designed redundantly to mitigate faults. The underlying platform implements a variant of the time-triggered architecture (TTA) [50], which provides precise timing and reliability guarantees. Safety-critical tasks and messages execute according to strict precomputed schedules to ensure synchronization between replicated components and provide fault mitigation and management. Deployed software implementations of the control functions must pass strict certification requirements which impose constraints on the software as well as on the development process. The additional burden of design analysis required to establish safety increases cost and schedule, decreasing the flexibility of the development process.

In modern embedded control system designs, graphical modeling and simulation tools (e.g. Mathworks’ Simulink/Stateflow) represent physical systems and engineering designs using block diagram notations. Design work revolves around simulation and test cases, with code generated from models when the design team reaches particular schedule milestones. Control designs often ignore software design constraints and issues arising from embedded platform choices. At early stages of the design, platforms may be vaguely specified to engineers as sets of trade offs [67].

Software development uses Unified Modeling Language Computer-Aided Software Engineering (UML CASE) tools to capture concepts such as types, components, interfaces, interactions, timing, fault handling, and deployment. Software development work flows focus on source code creation, organization, and management, followed by testing and debugging on target hardware. Physical and environmental constraints are not usually represented by the tools. At best such constraints may be provided as documentation to developers.

Complete control system software designs rely on multiple aspects. Designers lack tools to model the interactions between the hardware, software, and the environment with the required fidelity. For example, software generated from a carefully simulated synchronous dataflow model of the controller
functions may fail to perform correctly when its functions are distributed over a shared network of processing nodes. Cost or availability considerations may force the selection of platform hardware that limits timing accuracy or data precision beyond originally designed bounds. None of the current design, analysis, or development techniques support comprehensive (i.e. multi-domain) validation of certification requirements to meet government safety standards. Model and code analysis tools must all be integrated to have the same semantic view of the design details.

**Overview**

We aim to create a Domain Specific Modeling Language (DSML) to address problems of design consistency across the entire development flow for a distributed embedded control system design. Often, the best solutions involve iterating the design cycle as problems are discovered or problem understanding increases. Our DSML captures the relationships between concepts in the different design domains described, and supports the integration of analysis tools and code generation.

**High-Confidence Design Challenges**

We identify several specific challenges that arise because of inconsistencies between domains in a high-confidence embedded development project. Some of the challenges are fundamental, and others arise because of our attempts to use models to resolve consistency problems.

1. Controller, software, and hardware design domains are highly specialized and often conceptually incompatible. Sharing model artifacts between designers in different domains can lead to consistency problems in engineering solutions or implementations based on incomplete or faulty understanding of design issues. Current state of the art resolves differences in understanding by reviewing many of the details in numerous meetings and personal discussions. Manual reconciliation of issues occurs as individual designers receive assignments to modify and correct the design. In the worst cases serious incompatibilities are not discovered until very late in the design cycle, leading to project overruns and cancellations[4]. Several large modeling tool projects (for example, AADL [14] and Topcased[21]) work to integrate tools from independent research and development teams into a common design environment featuring a standardized
modeling language. Resolution of semantic consistency between integrated tools to improve design efficiency is a serious issue in such efforts.

2. Incompatibilities between models and assumptions in different design domains create a related problem. For example, controller design properties which are verified using simulation models may no longer be valid when the design becomes software in a distributed processing network. Currently control designers use conservative performance margins to avoid rework when performance is lost due to deployment on a digital platform.

3. Long development, deployment, and test cycles limit the amount of iterative rework that can be done to get a correct design. If a particular design analysis is costly or time-consuming, the team cannot afford to iterate the design from its early stages in order to resolve problems. Currently high-confidence design requires both long schedules and high costs.

4. Automating steps in different design and analysis domains for the same models and tools requires a consistent view of inferred model relationships across multiple design domains. If integrated tools have different views of the model semantics, then their analyses are not valid when the results are integrated into the same design. Therefore, all of the tools used in the design process must have a consistent view of design details. Explicitly reconciling semantics between formalisms and tools is costly and time-consuming. Often the effort cannot be justified outside of academic research unless the results are applicable to numerous designs.

5. As our research explores new directions in high-confidence design, modification of the ESMoL meta-model (language specification) creates maintenance problems for ESMoL models and for interpreter code that translates them into analysis artifacts and generated code. We would like to isolate interpreter development from the language to a degree in order to allow the ESMoL language to evolve with our research. ESMoL models can be updated to new versions of the language using features built into the tools, but nothing exists yet to handle those problems for interpreter code.
Model-Integrated Solutions

We propose a suite of tools that aim to address many of these challenges. Currently under development at Vanderbilt’s Institute for Software Integrated Systems (ISIS), these tools use the Embedded Systems Modeling Language (ESMoL), which is a suite of domain-specific modeling languages (DSML) to integrate the disparate aspects of a safety-critical embedded systems design and maintain proper separation of concerns between control engineering, hardware specification, and software development teams. The Embedded Systems Modeling Language (ESMoL) encodes in models the relationships between controller functions specified in Simulink, software components that implement those functions (i.e. dataflow, messaging interfaces, etc.), and the hardware platform on which the software will run. Many of the concepts and features presented here also exist separately in other tools. We describe a model-based approach to building a unified model-based design and integration tool suite which has the potential to go far beyond the state of the art.

1. The ESMoL language and tools provide a single multi-aspect embedded software design environment so that modeling, analysis, simulation, and code generation artifacts are all clearly related to a single design model. We aim to incorporate models appropriate to the different design domains in a consistent way using the Model-Integrated Computing (MIC) approach discussed below. ESMoL models use language-specified relations to associate Simulink control design structures with software and hardware design concepts to define a software implementation for controllers. Further, ESMoL is a graphical modeling language which integrates into existing Simulink-based control design work flows [68].

2. ESMoL models include objects and parameters to describe deployment of software components to hardware platforms. Analysis artifacts and simulation models generated from ESMoL models contain representations of the behavioral effects of the platform on the original design. We include platform-specific simulations to assess the effects of distributed computation on the control design [69].

3. ESMoL’s integrated analysis, simulation, and deployment capabilities can shorten design cycles. The ESMoL tool suite includes integrated scheduling analysis tools which converge quickly
in most cases ([70]) so that static schedules can be calculated in rapid design and simulation cycles. We include automatic generation of platform-specific task configuration and data communications code in order to rapidly move from modeling and analysis to testing on actual hardware.

4. ESMoL uses a two-stage interpreter architecture in order to integrate analysis tools and code generators. The first stage resolves any inferred model relationships from ESMoL models into a model in an abstract language (ESMoL_Abstract), much in the same way that a parser creates an abstract syntax tree for a program under compilation. The ESMoL design language allows relational inference where appropriate in order to make the user experience more productive. The Stage 1 interpreter resolves object instances, parameters, and relations, and stores them in an ESMoL_Abstract model. Model interpreters for analysis and generation use this expanded model to guarantee a consistent view of the relationships and details, and to share code efficiently in an integrated modeling tool development project. The two-stage approach also isolates the interpreter code from the structure of the ESMoL language. Changes to the language are principally isolated from the interpreter code by the first stage transformation.

5. We generate analysis models and code from the intermediate language using simple template generation techniques[70]. Round-trip incorporation of calculated schedule analysis results back into the ESMoL model helps to maintain consistency as models pass between design phases.

Fig. 6 depicts a design flow that includes a user-facing modeling language for design and an abstract intermediate language for supporting interpreter development and maintenance. During design, a software modeler imports an existing Simulink control design into the Generic Modeling Environment (GME) [71], configured to edit ESMoL models (Step 1). The modeler then uses the dataflow models imported from Simulink to specify the functions of software components which will be used to implement the controllers. These component specifications represent synchronous dataflow models that are realized as C code calls, and which are extended with interfaces defining input and output message structures for data distribution. We also specify the mapping from dataflow
Figure 6: Flow of ESMoL design models between design phases.
I/O ports to and from fields in the messages (Step 2). Designers specify the hardware topology for a time-triggered distributed processing network using another integrated design language (Step 3). A modeler instantiates component instances to create multi-aspect models where logical dependencies, hardware deployment, and timing models can be specified for the software architecture (Steps 4 and 5).

A completed model is transformed (via the Stage 1 transformation) into a model in the ESMoL_Abstract language, resolving all implied relationships and structural model inferences (Step 6). Model interpreters for design analysis (in this case calculating time-triggered schedules) are integrated using the Stage 2 model transformation from ESMoL_Abstract models to analysis specifications (Step 7). Another model interpreter imports results from the analysis (in this case, scheduled start times) back into the ESMoL_Abstract and ESMoL models (Steps 8 and 9). Finally, designers can also create platform-specific simulations and generate deployable code using the Stage 2 transformation (Step 10).

In a later section we discuss the relationship between the behavior represented by the original Simulink model and the behaviors represented by ESMoL and ESMoL_Abstract (Steps 5 and 10). ESMoL provides a great deal of modeling flexibility, as subsets of the Simulink model are used in Step 2 to define software components. These subsets can be replicated to model redundant computation networks, for example. In Step 4 they are aggregated to define dataflows, and then partitioned to define deployment of those dataflows. With all of the language flexibility provided, we need to ensure that the synchronous semantics of the original Simulink model are preserved in the distributed implementation in order to ensure that the inherent correctness properties (functional determinism, timing determinacy, and deadlock freedom) are also preserved.

The illustrated design flow represents only a single iteration in the overall development work flow to be discussed later. In the sequel we will use the expression design flow to indicate the work of modeling, analyzing, and generating code for a single design. Development flow will indicate the macro-level iterative development process which includes one or more design flow iteration.
Figure 7: Platforms. This metamodel describes a simple language for modeling the topology of a time-triggered processing network.

**Tools and Techniques**

The models in the example and the metamodels described below were created using the ISIS Generic Modeling Environment tool (GME) [71]. GME allows language designers to create stereotyped UML-style class diagrams defining metamodels. The metamodels are instantiated into a graphical language, and metamodel class stereotypes and attributes determine how the elements are presented and used by modelers.

The Model-Integrated Computing (MIC) approach[5] builds up DSMLs by creating specific sub-languages to capture concepts and relationships for different facets of the design domain, and then integrating those sublanguages into a common modeling language by precisely specifying the structural relationships between those sublanguages. In a GME metamodel a sublanguage is called a paradigm. We will use the terms sublanguage, language, and paradigm interchangeably. Confusion is resolved by explicitly naming the paradigms involved in the discussion.

The GME metamodeling syntax may not be entirely familiar to the reader, but it is well-documented in Karsai et al [71]. Class concepts such as inheritance can be read analogously to UML. Class aggregation represents containment in the modeling environment, though an aggregate
element can also be flagged as a port object. In the modeling environment a port object will also be visible at the next higher level in the model hierarchy, and available for connections. One unique notation in MetaGME (the GME modeling language for creating modeling languages) is the dot used for relating an association class to its endpoint connection classes. For example, the dot between the Connectable class and the Wire class (Fig. 7) represents a line-style connection in the modeling environment. One other useful concept from a GME metamodel is the reference. A reference object appears in the Metamodel specification associated with another class. This allows the modeler to create an object with the same interface (port structure) as the associated class, but which actually refers to the original object, much in the same way that a pointer refers to a different object in memory in a computer program.

Another key technology used in the ESMoL tool suite is the GReAT model transformation language (and its associated code generation tools)\cite{72}. The ESMoL suite contains a pair of platform-independent code generators for Simulink and Stateflow models. The transformations take Simulink and Stateflow blocks, and create equivalent models in another language (SFC) that corresponds to an abstract syntax graph for fragments of C code. Functional code generation proceeds by simply traversing and printing the SFC models. Other generators use the UDM C++ modeling API \cite{73} to create code implementing the platform-specific code to wrap functions as tasks, define communication messages structures, and configure a time-triggered virtual machine to execute the generated code. These generators as well as generators for platform-resimulation models are described elsewhere (see Porter et al \cite{68}, Thibodeaux \cite{74}, and Hemingway et al \cite{69} for details).

Platform-based design partitions design frameworks into designer-supplied components and platform-provided services\cite{67}. High-confidence systems require services and guarantees for correct and efficient execution such as real-time execution, data distribution, and fault tolerance. Platform-based design allows the construction of complex systems by facilitating reuse over common execution behaviors. The platform also defines a formal model of computation (MoC) \cite{75}, which predicts how the concurrent objects of an application interact (i.e. synchronization and communication). We use an implementation of the time-triggered architecture as a platform layer in order to reduce timing variances in sensing, actuation, and distributed data communications \cite{50,74}. The central idea of
the time-triggered architecture is to provide deterministic and fault-tolerant synchronous execution in order to ensure the consistent behaviors of distributed replicas of controller components.

**The ESMoL Languages**

To motivate our description of the facets of ESMoL, we focus on an actual control design model for the Starmac quadrotor helicopter [76][77]. Fig. 8 depicts its control architecture, consisting of two nested control loops. From left to right in the diagram, the *Input Filters* restrict the input trajectory commands to prevent maneuvers beyond the physically safe limits of the helicopter. The *Outer Loop* PD controller takes the requested position reference and the position data from the sensors, and calculates the attitude required for the quadrotor to achieve the requested change in position. *Saturation* is another limiter to ensure that the commanded attitude actuation is realizable. The *Inner Loop* PD controller takes the attitude command from the *Outer Loop* and measured attitude data, and calculates the motor thrusts required to achieve the commanded attitude. *Motor Compensator* filters the thrust commands to account for response delays in the motors which drive the rotors. Finally, the *Dynamic Model* describes the physical behavior of the helicopter, including the imprecision introduced by the sensors which measure position and attitude. The ESMoL model examples given below come from the design model for the quadrotor, except where noted.

**Requirements Analysis (RA)**

Formal requirements modeling offers great promise, but in ESMoL requirements modeling is still in conceptual stages. Informally, we require stability of the software-implemented closed-loop control
system over the full range of possible inputs, and satisfaction of the calculated timing constraints
(task release times and deadlines).

**Functional Design (FD)**

In ESMoL, functional specifications for components can appear in the form of Simulink/Stateflow
models or as existing C code snippets. ESMoL does not support the full semantics of Simulink. In
ESMoL the execution of Simulink data flow blocks is restricted to periodic discrete time, consistent
with the underlying time-triggered platform. This also restricts the type and configuration of blocks
that may be used in a design. Continuous integrator blocks and sample time settings do not have
meaning in ESMoL. C code snippets are allowed in ESMoL as well. C code definitions are limited
to synchronous, bounded response time function calls which will execute in a periodic task with a
fixed amount of memory.

An automated importer constructs an ESMoL model from a Simulink control design model. The
new model is a structural replica of the original Simulink model, only endowed with a richer software
design environment and tool-provided APIs for navigating and manipulating the model structure
in code. The Simulink and Stateflow sublanguages of our modeling environment are described
elsewhere[78]. The ESMoL language evolved from another DSML known as ECSL-DP. They share
many concepts, but ESMoL departs from many of the modeling structures previously described by
Neema in order to increase the flexibility and generality of the language.

**Component Design (CD)**

In the component design phase (CD) we specify software interfaces for the functions which will run
in the distributed controller network. A component type has a unique name (i.e. *InnerLoop*), and
information to find or generate its implementation in C (in this case, the file name and model path to
the Simulink subsystem “QuadRotor/STARMAC/InnerLoop”). A component specification contains
a reference to a Simulink subsystem, as well as references to message structure objects. The message
structure objects will represent message types, and each reference from a component definition
represents an interface through which that message is sent or received. Internally, the direction of
Figure 9: Quadrotor component types model from the *SysTypes* paradigm.

the connection from the message reference to the ports on the Simulink object determine whether the port sends or receives. We do not allow multi-directional message transfers on the same interface. When the component is instantiated in the design model (e.g., in the logical architecture diagram described below) the message references specified here will appear as ports on blocks representing the instance. Connections to and from those ports represent the transfer of an instance of that data message into or out of the component instance.

Fig. 9 shows an example of a model from the component interface definition language. Message fields and their sizes are specified here, as well as component implementations and interfaces. These specifications define software component types in an ESMoL model, which are instantiated and assigned to hardware in the architecture and deployment models, respectively. The quadrotor model has four different component types (each instantiated once) and six message types (instantiated as the ports objects appearing on the component instances later in the design). The breakout inset in the figure shows the internals of the *DataHandler* component specification. The *sensor_convert* subsystem block in the center is a reference to a Simulink block specifying the data conversions that transform raw sensor data into scaled, formatted data for use by the controller blocks.
The blocks on the outer edges of the figure (Fig. 9) are references to messages defined at the top level of the system types model. On the left is the raw data message from the sensors. On the right are the attitude data message (for local consumption by the inner loop), and the position data message (sent remotely to the outer loop). The three message reference blocks in the inset appear as ports on the DataHandler block (top left in the figure). Inside the component type definition, ports on the message objects correspond to C structure fields. The field types are inferred from the data types imported from the connected Simulink signal port objects. The connections between the message ports and the Simulink reference block ports describe the direction and details of data flow between the implemented message structures and the specified functional block.

Fig. 10 portrays the SystemTypes sublanguage, which encodes these structures and relations. Components of different types (here Simulink block references or C code blocks) specify the component functions. Message references (MessageRef objects) define interfaces on the components, and ports on message objects (MsgPort objects) represent message data fields as in the DataHandler
example. The Input and Output port classes are typed according to the implementation class to which they belong (i.e., either Simulink signal ports or C function arguments). The connections between the block reference and the MsgPort objects describe the details required to marshal and demarshal the data fields in the messages for use by the specified function. Synchronous, periodic, discrete-time Simulink blocks and bounded-time synchronous C function calls are compatible at this modeling stage, because their model elements both represent the code that will finally implement the functions. These units are modeled as blocks with ports, where the ports represent parameters passed into and out of C function calls. The Trigger and Event types are not discussed here, as they relate to future work in the ESMoL tool suite.

**Hardware Architecture (HwA)**

Fig. 11 illustrates the example platform model. The quadrotor architecture is deployed to a small embedded processor assembly manufactured by Gumstix, Inc. The outer loop position control is handled by an Intel PXA ARM processor (the Gumstix board), and attitude control and vehicle I/O are handled by an Atmel Atmega128 AVR processor (the Robostix board). The I/O occurs over serial connections to the sensors and motor actuators. The serial devices reside within the processor, and are modeled in the diagram as objects connecting the input and output ports on the processor to the object representing the plant dynamics. The two processors communicate via a synchronous \( I^2C \) bus which runs a software emulated time-triggered protocol.
A simple platform definition language (Fig. 7) contains relationships and attributes describing
time-triggered networks. The models contain network topology and parameters to describe behavioral
quantities like data rates and bus transfer setup times. Platforms are defined hierarchically
as hardware units with ports for interconnections. Primitive components include processing nodes
and communication buses. Behavioral semantics for these networks come from the underlying time-
triggered architecture. The time-triggered platform provides services such as deterministic execution
of replicated components and timed message-passing. Model attributes for hardware also capture
timing resolution, overhead parameters for data transfers, and task context switching times.

Architecture Language

Logical architecture, deployment, and timing/execution models represent different design aspects
for the same set of component instances. GME allows us to define the language in such a way that
these three model aspects are simply different views of the same set of model elements. Together,
the information in the three aspects define a model which is complete with respect to scheduling
analysis, platform-specific simulation, and code generation.

System design models defined in the architecture language do not necessarily represent complete
designs. For simple designs (such as the quadrotor example) a single architecture model can cap-
ture all of the details of the software model. More complex designs require an additional layer of
organization which is not described here. It suffices to say that designs represented in the ESMoL
Architecture language can be considered as fragments which can be assembled into more complex
structures. This is an active area of research for our ESMoL modeling efforts, as the higher-level
architecture models should also account for fault modeling, evaluation, and performance issues.

• Logical Software Architecture (SwA) Aspect Fig. 12 portrays an ESMoL model ex-
ample specifying logical data dependencies between quadrotor software component instances,
independent of their distribution over different processors. The software architecture model
describes the logical dataflow dependency relationships between component instances. Seman-
tics for SwA Connections are those of task-local synchronous function invocations (with shared
memory messaging) or message transfers between remote tasks using time-triggered communication. In this model the interpretations for the dependency links have not been specified. Those details appear in the deployment model.

For the quadrotor, the RefHandler and DataHandler components receive and process data from the sensors. They pass their formatted data to the respective control blocks. The OuterLoop calculates an attitude reference to achieve the requested position. The InnerLoop issues thrust commands to achieve the requested attitude.

In a design model, creation of a (GME) reference object to one of the component types corresponds to instantiation. Fig. 13 illustrates this idea. Using the same controller components along with a few new components to implement voting logic, we have specified the logical architecture for a triply-redundant version of the quadrotor model. Each ESMoL component type is used multiple times in a single design, expanding the model structure far beyond the size and scope of the original Simulink design. This particular model diagram is only shown to illustrate the instantiation mechanism.

- **Deployment Models (SY, DPL)** Fig. 14 displays the deployment model – the mapping of software components to processing nodes, and data messages to communication ports. Two of the four components are mapped to each of the two processors. For the quadrotor, the RefHandler and OuterLoop tasks run on the Gumstix processor. The InnerLoop and DataHandler tasks run on the Robostix processor. RefHandler receives position commands from a socket connection. DataHandler receives sensor data from a UART channel (a processor port
Figure 13: Triply-redundant quadrotor logical architecture. This is not part of the actual quadrotor model, and is only given for illustration.

Figure 14: Quadrotor architecture model, Deployment aspect.
in the model diagram). Position and attitude data are exchanged over the time-triggered bus, so the corresponding message ports are connected to bus channel objects on their respective processors. *InnerLoop* sends thrust commands through a UART channel, hence the connection to the appropriate processor port.

In the figure the dashed connection from a component to a node reference represents an assignment of that component to run as a task on the node. The port connections represent the hardware channel through which that particular message will travel. Remote message dependencies are assigned to bus channels on the node. Local data dependencies are not specified here, as they are represented in the logical architecture. *IChan* and *OChan* port objects on a node can also be connected to message objects on a component. These connections represent the flow of data from the physical environment through sensors (*IChan* objects) or the flow of data back to the environment through actuators (*OChan* objects). Model interpreters use deployment models to generate platform-specific task wrapping and communication code as well as scheduling problem specifications.

The metamodel in Fig. 15 illustrates the classes and relationships for both the logical architecture connections and the deployment mapping. GME metamodels have a separate visualization aspect that allows us to define aspects in ESMoL and indicate which classes and connections should be visible in each aspect. *ComponentRef* objects are software component instances, and are visible in both aspects. In the logical architecture aspect, *Dependency* connectors define
message transfers between component instance ports. The ports represent interfaces for each component instance. For the deployment aspect we add NodeRef objects (node references) and connectors (ComponentAssignment and CommMapping) to identify the mapping of tasks and messages to the platform model.

The deployment aspect captures the assignment of component instances as periodic tasks running on a particular processor. In ESMoL a task executes on a processing node at a single periodic rate. All components within the task execute synchronously. Data sent between tasks take the form of messages in the model. For data movement, the runtime provides logical execution time semantics found in time-triggered languages such as Giotto [79] – message transfers are scheduled after the deadline of a sending task, but before the release of the receiving tasks. Tasks never block, but execute with whatever data is available for each period.

- **Timing Models** Fig. 16 shows the quadrotor timing and execution model, where the designer attaches timing parameter blocks (of type TTExecInfo) to components and messages. TTExecInfo block configuration parameters include execution period and worst-case execution time. In the quadrotor model all task and message transfers are timed. The quadrotor data network runs at a rate of 20ms. Particular timings for tasks and data transfers will be discussed below in the evaluation discussion.
The timing sublanguage (Fig. 17) allows the designer to specify component execution constraints. Individual components can be annotated with timing objects that indicate whether they should be executed strictly (i.e., via statically scheduled time-triggered means), or as periodic real-time or sporadic tasks. Messages are similarly annotated. The annotation objects contain parameters such as period and worst-case execution time that must be given by the designer. Automated scheduling analysis fills in the schedule fields.

The execution model also indicates which components and messages will be scheduled independently, and which will be grouped into a single task or message object. The time order of the message writer and readers are enforced by the static schedule. The locality of a message transfer is specified in the logical architecture and deployment aspects. In the case of processor-local data transfers, transfer time is neglected – reads and writes occur in locally shared memory. After a static schedule has been calculated, task and message release times are also stored in the timing objects.

Behavior of the deployed software components depends on the execution times of the functions on the platform, the calculated schedule, and coordination between distributed tasks. The calculated static execution schedule can be used to simulate the control design with additional delays to assess the impact of the platform on performance.
Integrating Tools with ESMoL

Figure 6 depicts a design flow that includes a user-facing modeling language for design and an abstract intermediate language for supporting interpreter development and maintenance. A completed ESMoL model is transformed (via the Stage 1 transformation, Step 6 in the figure) into a model in the ESMoL_Abstract language, where all implied relationships and structural model inferences have been resolved. Model interpreters for calculating time-triggered schedules, creating platform-specific simulations, and generating deployable code are integrated using the Stage 2 transformation.

Rather than designing a user-friendly graphical modeling language and directly attaching translators to analysis tools, we created a simpler abstract intermediate language whose elements are similar to those of the user language. The first model transformation flattens the user model into the abstract intermediate form, translating parameters and resolving special cases as needed. Generators for code and analysis are attached to the abstract modeling layer, so the simpler second-stage transformations are easier to maintain, and are isolated from changes to the user language.

In the model integrated computing approach, domain specific modeling languages represent different aspects of the design, with the aim of consistently integrating different concepts and details for those design aspects and integrated analysis tools. Our tools enforce a single view of structural inference in the design model. We will cover some of the transformation details to illustrate this concept. This approach can be considered as an implementation of the tool integration ideas in [80], but with variations of the details included in the design language.

Stage 1 Transformation

Stage 1 translates ESMoL models into an abstract intermediate language that contains explicit relation objects that represent relationships implied by structures in ESMoL (Fig. 18). This translation is similar to the way a compiler translates concrete syntax first to an abstract syntax tree, and then to intermediate semantic representations suitable for optimization. Stage 1 was implemented using the UDM model navigation API, and written in C++. The ESMoL_Abstract target model is the source for the transformations implemented in Stage 2.
Table 4: Acquisition relation transformation details.

\[
CA_{idN} = \{(obj_{Node}, obj_{CompInst}) \mid id(obj_{Node}) = id_N\}
\]
\[
AC_{idCh} = \{(obj_{IChan}, obj_{MsgInst}) \mid id(obj_{IChan}) = id_{Ch}\}
\]
\[
NC_{idN} = \{(obj_{Node}, obj_{IChan}) \mid id(obj_{Node}) = id_N \land parent(obj_{IChan}) = obj_{Node}\}
\]
\[
CC = \{(obj_{CompInst}, obj_{MsgInst}) \mid parent(obj_{MsgInst}) = obj_{CompInst}\}
\]

Figure 19: Acquisition relation in ESMoL Abstract, representing the timed flow of data arriving from the environment.

Figure 18: Stage 1 Transformation.
Each analysis translation works from a single view of the design model, simplifying the implementations of tool-specific translations. As an example, consider the model shown in Fig. 12. Component `DataHandler` sends data messages to the other two components, as denoted by the dependency arrows. The deployment view (Fig. 14) shows that each component executes on a different processor. Locally, the port object on each component (in both diagrams) represents the component’s view of the data message sent over the wire. The solid connections in the deployment diagram indicate which device on the processing node will be used to transfer the data. Specified messages will participate in processor-local synchronous data flows, or time-triggered exchanges over the network. All of these connections and entities are related to a single semantic message object, which is related to other elements in different parts of the user model (see the FormattedData message in Fig. 23). The execution aspect contains timing information objects, which provide information for fully specifying the various data transfers.

The first stage transformation checks constraints to ensure that each object is used correctly throughout the design, ensuring well-formedness. The Stage 1 transformation then reduces this complex set of relations to a single message object with relations to the other objects that use it. Timing parameters from the platform model are used to calculate a behavioral model for messages and components, including component start times, message transfer times, and the duration of each message on the bus.

We describe here some of the transformations of user-facing ESMoL language objects and relations to a more compact set of relations that simplify generation of design artifacts from the model. The most direct example of such a semantic assumption is the single-message abstraction. Data transfers between the functional code and the message fields must be compatible. We enforce compatibility both by constraint checking, and by the use of a single ESMoL Abstract message instance object for all participants in the data interchange. The `Signal` object in the abstract graph represents the transfer of a single datum to or from the message. For simplicity and clarity we will not show the `Signal` objects in the diagrams, as they are numerous.
The transformations described here capture different forms of the single-message transformation. This is not a complete description of the entire first stage transformation, but provides a representative subset for illustration.

In the formal descriptions below, \( \text{Obj}_{\text{Type}} \) (capitalized) is the set of objects of type \( \text{Type} \), and \( \text{obj}_{\text{Type}} \) (lowercase) is an instance from that set. We also use two functions \( \text{id} : \text{Obj}_{\text{Type}} \rightarrow \mathbb{Z}^+ \) for a unique identifier of an object, and \( \text{parent} : \text{Obj}_{\text{Type}1} \rightarrow \text{Obj}_{\text{Type}2} \) to find the parent (defined by a containment relation in the model of an object). The parent relation is unique.

**Acquisition: From the Environment to Data**

In ESMoL\_Abstract Acquisition objects relate all of the different model entities (and therefore, their design parameters) that participate in the collection of data from an input device such as an analog to digital converter or serial link. The Stage1 transformation enforces certain cardinality constraints to ensure the validity of this transformation – for example, each message instance is related to exactly one sender and possibly multiple receivers. A message relationship can be implied by different types of connections in ESMoL, so Stage1 must determine that only one such relationship exists.

The ESMoL relations shown in Table 4 are described as follows:

- **CA ComponentAssignment**: (the dashed connection shown in Fig. 14) assigns a task to run on a particular processor (\( \text{id}_N \)).

- **AC AcquisitionConnection**: (the directed connection from processor object ports to component message ports) assigns a hardware input peripheral data channel (modeled as an object of type \( I\text{Chan} \)) to a data-compatible message structure in the component.

- **NC**: Containment relationship of the channel object (port) in the Node object.

- **CC**: Containment of the message instance object (port) in the component instance object.

The metalanguage for ESMoL\_Abstract captures the structural semantic reductions shown in Table 4 in a compact form (see Fig. 19), so that all of the consumers of the input data get the same consistent structural view of the model. This transformation takes the ESMoL objects described in the left column of the table and produces a single relation for each collection representing an
Specified ESMoL Relation Sets

<table>
<thead>
<tr>
<th>Relation Set</th>
<th>Specification</th>
</tr>
</thead>
<tbody>
<tr>
<td>CA&lt;sub&gt;id_N&lt;/sub&gt;</td>
<td>( {(\text{obj}N_{node}, \text{obj}CompInst) \mid \text{id}(\text{obj}N_{node}) = \text{id}_N} )</td>
</tr>
<tr>
<td>AC&lt;sub&gt;id.Ch&lt;/sub&gt;</td>
<td>( {(\text{obj}OChan, \text{obj}MsgInst) \mid \text{id}(\text{obj}OChan) = \text{id}_N} )</td>
</tr>
<tr>
<td>NC&lt;sub&gt;id.N&lt;/sub&gt;</td>
<td>( {(\text{obj}N_{node}, \text{obj}OChan) \mid \text{id}(\text{obj}N_{node}) = \text{id}<em>N \land \text{parent}(\text{obj}OChan) = \text{obj}N</em>{node}} )</td>
</tr>
<tr>
<td>CC</td>
<td>( {(\text{obj}CompInst, \text{obj}MsgInst) \mid \text{parent}(\text{obj}MsgInst) = \text{obj}CompInst} )</td>
</tr>
</tbody>
</table>

ESMoL<sub>Abstract</sub> Relation

<table>
<thead>
<tr>
<th>Relation</th>
<th>Specification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Act</td>
<td>( {(\text{obj}MsgInst, \text{obj}CompInst, \text{obj}N, \text{obj}Ch) \mid (\text{obj}N, \text{obj}CompInst) \in \text{CA}<em>{id_N} \land (\text{obj}Ch, \text{obj}MsgInst) \in \text{AC}</em>{id.Ch} \land (\text{obj}N, \text{obj}OChan) \in \text{NC}_{id.N} \land (\text{obj}CompInst, \text{obj}MsgInst) \in \text{CC}} )</td>
</tr>
</tbody>
</table>

Table 5: Actuation relation transformation details.

Figure 20: Actuation relation in ESMoL<sub>Abstract</sub>, representing the timed flow of data back into the environment.

ESMoL<sub>Abstract</sub> data acquisition specification. The modeling tools provide a programming interface for traversing, reading, and editing the models. The collected relations are also more efficiently processed by synthesis interpreters, as they avoid extra traversals to gather the objects.

**Actuation: From Data to the Environment**

The transformation to an Actuation object is nearly identical to that of the Acquisition transformation, but the data direction, cardinalities, and types involved are different. The chief difference is that actuation objects can only have one associated task, where acquisition data may be broadcast to multiple tasks. Table 5 gives the details of the transformation from relations in ESMoL to the actuation relation in ESMoL<sub>Abstract</sub>. Fig. 20 shows the structure of the resulting classes in ESMoL<sub>Abstract</sub>.
**Specified ESMoL Relation Sets**

| CA_{id_N} = \{(\text{obj}_\text{Node}, \text{obj}_\text{CompInst}) | id(\text{obj}_\text{Node}) = \text{id}_N\} |
| LD_{id_1} = \{(\text{obj}_\text{MsgInst}_1, \text{obj}_\text{MsgInst}_2) | id(\text{obj}_\text{MsgInst}_2) = \text{id}_1\} |
| LD_{TC} = \{(\text{obj}_\text{MsgInst}_j, \text{obj}_\text{MsgInst}_{j+1}) | in the sequence \((\text{obj}_\text{MsgInst}_1, \text{obj}_\text{MsgInst}_2) \in LD_{id_1}, \(\text{obj}_\text{MsgInst}_2, \text{obj}_\text{MsgInst}_3) \in LD_{id_2}, \ldots\), \((\text{obj}_\text{MsgInst}_j, \text{obj}_\text{MsgInst}_{j+1}) \in LD_{id_j}\)\} |
| CC = \{(\text{obj}_\text{CompInst}, \text{obj}_\text{MsgInst}) | parent(\text{obj}_\text{MsgInst}) = \text{obj}_\text{CompInst}\} |

**ESMoL_Abstract Relation**

| Locals = \{(\text{obj}_\text{MsgInst}_1, \text{obj}_\text{CompInst}_1, \text{obj}_\text{MsgInst}_2, \text{obj}_\text{CompInst}_2, \text{obj}_N) | (\text{obj}_N, \text{obj}_\text{CompInst}_1) \in CA_{id_N}, \text{obj}_\text{CompInst}_2) \in CA_{id_N}, \text{obj}_\text{MsgInst}_1, \text{obj}_\text{MsgInst}_2) \in LD_{TC}, \text{obj}_\text{CompInst}, \text{obj}_\text{MsgInst}) \in CC\} |

Table 6: Local (processor-local) data dependency relation.

**Figure 21**: Local dependency relation in ESMoL Abstract, representing data transfers between components on the same processing node.

**Local Dependencies**: Data Movement within Nodes Local dependencies represent not only direct data dependencies between nodes on a particular processor, but also implied dependencies through remote data transfer chains starting and ending on the same processor. This is modeled as the set \(LD_{TC}\) of all pairs in the transitive closure of dependencies starting with the message instance \(obj_{\text{MsgInst}}\). The collected set of local dependencies (Locals) intersects this set with those message instances contained in components on the current processing node (i.e. from the set \(CA_{id_N}\)). Table 6 gives the transformation details.

**Bus Transfers**: Data Movement Between Nodes Bus transfers are slightly more complicated, as they involve two or more endpoints. Table 7 and Fig. 22 contain the details. The send
Specified ESMoL Relation Sets | ESMoL Abstract Relation
---|---
$CA_{id_N} = \{(obj_{Node}, obj_{CompInst}) | \quad \text{id}(obj_{Node}) = id_N\}$

$AC_{id_{Ch}} = \{(obj_{MsgInst}, obj_{BChan}) | \quad \text{id}(obj_{BChan}) = id_{Ch}\}$

$NC_{id_N} = \{(obj_{Node}, obj_{BChan}) | \quad \text{id}(obj_{Node}) = id_N \quad \land \quad \text{parent}(obj_{BChan}) = obj_{Node}\}$

$CC = \{(obj_{CompInst}, obj_{MsgInst}) | \quad \text{parent}(obj_{MsgInst}) = obj_{CompInst}\}$

$Trn = \{(obj_{MsgInst}, obj_{CompInst}, \quad obj_{N}, \quad obj_{Ch}) | \quad (obj_{N}, obj_{CompInst}) \in CA_{id_N} \quad \land \quad (obj_{Ch}, obj_{MsgInst}) \in AC_{id_{Ch}} \quad \land \quad (obj_{N}, obj_{BChan}) \in NC_{id_N} \quad \land \quad (obj_{CompInst}, obj_{MsgInst}) \in CC\}$

Table 7: Transmit relation transformation details. This represents the sender side of a remote data transfer between components.

Specified ESMoL Relation Sets | Semantic Construct
---|---
$CA_{id_N} = \{(obj_{Node}, obj_{CompInst}) | \quad \text{id}(obj_{Node}) = id_N\}$

$AC_{id_{Ch}} = \{(obj_{BChan}, obj_{MsgInst}) | \quad \text{id}(obj_{BChan}) = id_{Ch}\}$

$NC_{id_N} = \{(obj_{Node}, obj_{BChan}) | \quad \text{id}(obj_{Node}) = id_N \quad \land \quad \text{parent}(obj_{BChan}) = obj_{Node}\}$

$CC = \{(obj_{CompInst}, obj_{MsgInst}) | \quad \text{parent}(obj_{MsgInst}) = obj_{CompInst}\}$

$Rcv = \{(obj_{MsgInst}, obj_{CompInst}, \quad obj_{N}, \quad obj_{Ch}) | \quad (obj_{N}, obj_{CompInst}) \in CA_{id_N} \quad \land \quad (obj_{Ch}, obj_{MsgInst}) \in AC_{id_{Ch}} \quad \land \quad (obj_{N}, obj_{BChan}) \in NC_{id_N} \quad \land \quad (obj_{CompInst}, obj_{MsgInst}) \in CC\}$

Table 8: Receive relation transformation details.

Figure 22: Transmit and receive relations in ESMoL Abstract, representing the endpoints of data transfers between nodes.
Figure 23: Object diagram from part of the message structure example from Figs. 12 and 14.
and receive relations are modeled separately as they have different cardinalities (one sender and possibly multiple receivers). The platform-specific code generators produce separate files for each processor (recall that the network may be heterogeneous). Fig. 23 shows an example of the objects and parameters based on our design example. The object diagram is an instance of the abstract language constructs shown in Figs. 19, 21, and 22. The diagram depicts ESMoL_abstract relations of type Acquisition, LocalDependency, Transmits, and Receives. These objects are involved in collecting position data from the sensors (task DataHandler from data channel Robostix_UARTChan1), and then redistributing it locally to the InnerLoop task as well as remotely to the OuterLoop task through the bus channel interfaces on the Robostix and Gumstix nodes.

Stage 2 Transformation Outputs: Analysis Models and Code

Stage 2 generates analysis models and code from ESMoL_abstract models (Fig. 24). To perform the actual generation of analysis models and code, we use the CTemplate library[81] called from C++. The current Stage 2 interpreter is generally used in a particular sequence:

1. Generation of the scheduler specification.

2. Creation of a TrueTime simulation model.

3. Generation of platform-specific code using the FRODO virtual machine API.
Figure 25: Integration of the scheduling model by round-trip structural transformation between the language of the modeling tools and the analysis language.

We will cover details for generation of scheduling problem specifications and FRODO-specific code. The TrueTime code generation is documented elsewhere[69].

**Scheduling Problem Generation**

The control design models provide task period configurations, and either profiling or static analysis provides worst-case execution time parameters for each component instance. Data transfer rates and overhead parameters for communication buses are stored in the platform model. [70] describes the mapping of model structure, execution information, and platform parameters into actual constraint model details, extending earlier work on constraint-based schedule calculation[34]. The Gecode constraint programming tool [82] solves these constraints for task release and message transfer times on the time-triggered platform. The scheduling process guarantees that the implementation meets the timing requirements required by the control design process.

Fig. 25 portrays the steps a model transformation takes while distilling details from ESMoL and creating a scheduling problem model whose syntax represents the proper sets of behaviors. If the
schedule is feasible, task and message release time results are fed back into the ESMoL model as configuration parameters. We describe the steps indicated in the diagram here:

1. We start with a design model specified using ESMoL.

2. The two-stage transformation converts the model to an equivalent model in ESMoL_Abstract, and then invokes the templates to generate a scheduling problem specification.

3. We invoke the scheduling tool, which performs the following steps:

   (a) Parses the problem specification to import the model into the constraint generation environment.

   (b) Calculates the hyperperiod length to determine the number of instances required for each task and message.

   (c) Translates task and message instance relationships into constraints in Gecode (as described in [70]).

   (d) Solves the constraint problem, possibly indicating infeasibility.

   (e) If a valid schedule results, it is written out to a file.

4. The results are imported into the ESMoL model and written to the appropriate objects.

Table 9 contains the distributed schedule specification for our quadrotor example, including the following elements:

- **Resolution** (seconds) specifies the size of a single processing tick for the global schedule. This should correspond to the largest measurable time tick (quantum) of the processors in the network. All tasks and messages in the schedule timeline are discretized to this resolution.

- **Proc** specifies a processing node. Parameters are name, processor speed (Hz), and message send/receive overhead times (these default to zero seconds if unspecified). Processor names must be unique.

- **Comp** (or task) belongs to the most recently specified processor. A component is characterized by its name, period, and worst-case execution time (WCET) (both in seconds). We do not address the manner in which the WCET is to be obtained.
Resolution 1ms

Proc RS 4MHz 0s 0s
Comp InnerLoop = 50Hz 1.9ms
Comp DataHandling = 50Hz 1.8ms
Comp SerialIn = 50Hz 1us
Comp SerialOut = 50Hz 1ms
Msg DataHandling.sensor_data_in 1B RS/SerialIn RS/DataHandling
Msg InnerLoop.thrust_commands 37B RS/InnerLoop RS/SerialOut
Msg DataHandling.ang_msg 1B RS/DataHandling RS/InnerLoop

Proc GS 100MHz 0s 0s
Comp RefHandling = 50Hz 1us
Comp OuterLoop = 50Hz 245us
Msg RefHandling.pos_ref_out 9B GS/RefHandling GS/OuterLoop

Bus TT_I2C 100kb 1.3ms
Msg OuterLoop.ang_ref 20B GS/OuterLoop RS/InnerLoop
Msg DataHandling.pos_msg 8B RS/DataHandling GS/OuterLoop

Table 9: Scheduling spec for the Quadrotor example.

<table>
<thead>
<tr>
<th>Resolution</th>
</tr>
</thead>
<tbody>
<tr>
<td>{{RESOLUTION}}</td>
</tr>
<tr>
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</tr>
<tr>
<td>Proc {{NODENAME}} {{NODEFREQ}} {{SENDHOHD}} {{RECVHOHD}}</td>
</tr>
<tr>
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</tr>
<tr>
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</tr>
<tr>
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</tr>
<tr>
<td>{{HOST_SECTION}}</td>
</tr>
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</tr>
<tr>
<td>{{MSG_SECTION}}</td>
</tr>
<tr>
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</tr>
<tr>
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</tr>
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</tr>
<tr>
<td>{{LATENCY_SECTION}}</td>
</tr>
</tbody>
</table>

Table 10: Stage 2 Interpreter Template for the Scheduling Specification

- **Bus** specifies a bus object, characterized by name, transfer speed (bits per second), and transfer overhead (also in seconds).

- **Msg** includes a name, byte length, sending task, and list of receiving tasks.

Task and message names are unique only within their scope (processor or bus, respectively). When used in other scopes they are qualified with their scope as shown (e.g. P3/T1). The timing constraints include the various platform overhead parameters. For example, once the message length is converted from bytes to time on the bus, we add the transfer overhead to represent the setup time for the particular protocol. Engineers must measure or estimate platform behavioral parameters and include them in models for the platform[67].
Scheduling specifications are created in the Stage 2 interpreter from the template shown in Table 10. The Stage 2 scheduler generation logic traverses the ESMoL Abstract model and fills in the structures which are used to fill in the template when the CTemplate generator is invoked. In CTemplate, each {{#...}} {{/...}} tag pair delimits a section which can be repeated by filling in the proper data structure in the code. The other tags {...} are replaced by the string specified in the generation code.

Producing the Proc and Comp lines from the model API is straightforward as the output mirrors the model hierarchy, so these lines require only simple traversals of the model. Each generated line uses parameters from the respective model object to fill in the blanks. The parameters are shown only in the generated output, though the object diagram in Fig. 23 illustrates a good example of parameter layout and disposition. In order to produce each Msg line, many relations must be collected (as shown in Table 6 and Fig. 21) and distilled into the right relationships. This requires more complex traversal code often involving multiple passes through the model objects. To write a new generator similar to this one, the developer uses the interpreter API and the transformed abstract syntax graph model. In the abstract language traversal we collect the LocalDependency objects and filter them by processor. Each LocalDependency object contains all of the information necessary to fill out the parameters in the template and create a new Msg line in the scheduler specification file (within the proper Proc scope).

While we do not list here the details related to the solution of scheduling specifications, it may be useful to document some of the scheduler limitations. More details regarding these limitations may be found in [70].

- We do not support preemptive scheduling of tasks or messages, as our runtime provides conflict-free task execution and data communication during nominal operation.

- The overhead parameters may be an overly simplistic model for some cases. Each processor and bus pair may have different parameters, depending on the bus type and the protocol used.
• We do not perform optimization on the schedule, so performance cost functions are not taken into account. For control problems where the execution time changes yield irregular performance changes, this is a more serious issue (see for example [83]).

Platform-Specific Code Generation

Time-triggered execution requires configuration with the computed cyclic schedule. Code generated for the virtual machine conforms to a particular synchronous execution strategy – each task reads its input variables, invokes its component functions, and writes its output variables. The schedule calculation assumes logical execution time semantics, where task input data is ready before task release, and output data is not assumed ready before the task completes[17]. Data structures describe the invocation times and configuration parameters for tasks and messages on each processor. Each message configuration instance also includes local buffer addresses where the timed communication controller in the virtual machine can store incoming and outgoing message data.

The generated code for the Quadrotor model in Table 11 was produced from the template description for the platform-specific code generator in Table 12. The FRODO virtual machine generation template brings together all of the ESMoLAbstract relations described in the earlier section. The template and generated code segment above correspond to the second-stage interpreter that creates the static schedule structures used by the virtual machine. The tasks, messages, and peripherals listed here come from the Acquisition, Actuation, Transmit, and Receive relation objects. The various connected objects are sorted according to schedule time, and then the template instantiation uses the object parameters to create the tables in a manner similar to that described for the scheduler specification generation above. The LocalDependency relations do not appear in this template. The scheduler creates constraints that must be satisfied for each local dependency, but local message transfers take place automatically in shared memory as tasks write to and read from processor-local message structures. Therefore, any valid task and message schedule will satisfy them. In a different part of the FRODO template, the local dependencies determine which message fields must be used as arguments to the component function calls (not shown here).
portTickType hp_len = 20;

task_entry tasks[] = {
    { DataHandling, "DataHandling", 4, 0 },
    { InnerLoop, "InnerLoop", 9, 0 },
    {NULL, NULL, 0, 0}
};

msg_entry msgs[] = {
    { 1, MSG_DIR_RECV, sizeof( OuterLoop_ang_ref ),
        (portCHAR *) & OuterLoop_ang_ref,
        (portCHAR *) OuterLoop_ang_ref_c, 7, 0, 0 },
    { 2, MSG_DIR_SEND, sizeof( DataHandling_pos_msg ),
        (portCHAR *) & DataHandling_pos_msg,
        (portCHAR *) DataHandling_pos_msg_c, 11, 0, 0 },
    { -1, 0, 0, NULL, NULL, 0, 0, 0 }
};

per_entry pers[] = {
    { 1, "UART", IN, 0, 0, sizeof( DataHandling_sensor_data_in ),
        (portCHAR *) & DataHandling_sensor_data_in,
        (portCHAR *) DataHandling_sensor_data_in_c, 2, NULL, 0, 0 },
    { 2, "UART", OUT, 0, 0, sizeof( InnerLoop_thrust_commands ),
        (portCHAR *) & InnerLoop_thrust_commands,
        (portCHAR *) InnerLoop_thrust_commands_c, 14, NULL, 0, 0 },
    { -1, NULL, 0, 0, 0, NULL, NULL, 0, NULL, 0, 0 }
};

Table 11: Generated code for the task wrappers and schedule structures of the Quadrotor model.
portTickType hp_len = {{NODE_hyperperiod}};

{{#SCHEDULE_SECTION}}
task_entry tasks[] = {
  {{#TASK}}
    { {{TASK_name}}, "{{TASK_name}}", {{TASK_startTime}}, 0},{{TASK}}
    {NULL, NULL, 0, 0}
  }

msg_entry msgs[] = {
  {{#MESSAGE_NAME}}
    { {{MESSAGE_index}}, {{MESSAGE_sendreceive}}, sizeof( {{MESSAGE_name}} ),
      (portCHAR *) & {{MESSAGE_name}},
      (portCHAR *) & {{MESSAGE_name}}.c, {{MESSAGE_startTime}},
      pdFALSE},
  }{{MESSAGE_NAME}}
    { -1, 0, 0, NULL, NULL, 0, 0}
  }

per_entry pers[] = {
  {{#PER_NAME}}
    { {{PER_index}}, "{{PER_type}}", {{PER_way}}, 0, {{PER_pin_number}}, sizeof( {{PER_name}} ),
      (portCHAR *) & {{PER_name}},
      (portCHAR *) & {{PER_name}}.c,
      {{PER_startTime}}, NULL},
  }{{PER_NAME}}
    { -1, NULL, 0, 0, 0, NULL, NULL, 0, NULL }
  }

}{{SCHEDULE_SECTION}}

Table 12: Template for the virtual machine task wrapper code. The Stage 2 FRODO interpreter invokes this template to create the wrapper code shown in Table 11.
Synchronous Semantics

We will briefly present a formal argument for the preservation of synchronous Simulink block firing orders as we use the Simulink blocks to define software components, their deployment to the hardware, and impose a time-triggered execution schedule on the design. Our semantic argument is only valid for synchronous data flow (SDF) specifications. We do not claim to represent the full generality of Simulink specifications, rather we restrict ourselves to dataflow graphs without conditional execution. Our graphs must contain only tasks that execute in periodic, discrete time, have no delay-free loops, all delay elements must be initialized with a data token, and initial block firing orders must include the outputs of the delay elements. This final assumption can be satisfied by considering the outputs of the delay elements as additional inputs to the component. Then all dependent blocks will be able to fire as early as necessary in the schedule. Our restrictions on execution are consistent with those required by the Mathworks Real-Time Workshop Embedded Coder product, which forces models to have fixed-step execution and task periods harmonic with the configured time step size for code generation.

Consider a synchronous acyclic graph $G = (V, E)$ representing the connectivity of a Simulink dataflow model, where edges abstract the transfer of data between blocks (without the data type information, data capacity, or multiplicity). Let $\text{exec} : G \rightarrow \mathbb{R}$ represent the task duration for vertices (obtained by analysis or measurement), and the communication message transfer time for edges.

Let $C_V \subseteq V \times \mathbb{Z}^{|V|}$ be the set of all possibly concurrent firing orders for the blocks represented by the set $V$ which respect the partial order specified by the edge set $E$ (i.e. $(v_1, v_2) \in E \Rightarrow c(v_1) < c(v_2) \forall c \in C_V$ where the pairs in $c$ are interpreted as functions on $V$). Note that $C_V$ should only be taken up to isomorphism, eliminating orderings that are equivalent.

Consider the synchronous execution of $G$, where the full graph is executed on a periodic schedule at instants $\{T_k\}, (k = 0, 1, \ldots, \infty)$, and completes each execution before the next cycle. For embedded code generation, Simulink requires models to execute with fixed-step semantics, so this is not an overly strong restriction.
Next, we allow manipulations of the dataflow graph $G$ as follows: Let $G' \subseteq G$ be a subgraph. Assume that $G' = (V', E')$ represents a well-formed functional dataflow. Let $C_{V'} \subseteq V' \times \mathbb{Z}^{|V'|}$ be a set of possible orderings for $V'$ created by restricting $C_V$ to the vertex set $V'$. All orderings in $C_V$ are also valid in $C_{V'}$, if we adjoin proper orderings from $C_{V-V'}$. All orderings in $C_{V'}$ are also synchronous orderings, as they respect the partial order defined by $G'$. We can also continue this construction for products. Let $G' \subseteq G$ and $G'' \subseteq G$, where we uniquely identify the vertex sets $V'$ and $V''$ so that $V' \cap V'' = \emptyset$. Then for $G' \times G''$ we have $C_{V'} \times C_{V''}$. Considering the concurrent execution of $G'$ and $G''$, both graphs execute synchronously if executed according to an order from $C_{V'} \times C_{V''}$.

Let $G_{s_1}, \ldots, G_{s_I}$ be subgraphs of a Simulink dataflow $G$. These represent ESMoL-specified dataflows. Let $G_s = \times_{i \in [1, I]} G_{s_i}$, where each vertex $v \in V_s$ is given a unique identity as above. Consider the product of the restricted orderings $C_{V_s} = \times_{i \in [1, I]} (V_s \times \mathbb{Z}^{|V_{s_i}|})$. Then the specified dataflow $G$ is synchronous if executed according to an order from $C_{V_s}$.

Consider the following partitions on $G_s$:

Let $C_P : V_s \to [1, P]$ assign component blocks to physical processors.

Let $C_T : V_s \to [1, N]$ assign components to computational tasks. We need to ensure that all components belonging to the same task also belong to the same processor ($\forall n \in [1, N], \exists p \in [1, P] : \{v \in V_s | C_T(v) = n\} \Rightarrow \{v | C_P(v) = p\}$.

Let $\{b_1, \ldots, b_B\}$ be a set of physical communication buses, $\{t_1, \ldots, t_N\}$ be the set of tasks, and $\{p_1, \ldots, p_P\}$ be the set of physical processors. Let $B_E = \{b_1, \ldots, b_B\} \cup \{t_1, \ldots, t_N\} \cup \{p_1, \ldots, p_P\}$. Let $C_E : E_S \to B_E$ represent the communication mode for each data message represented by a graph edge. Data can travel remotely (via a data bus $b_i$) or locally in shared memory (between components within a task $t_j$ or between tasks on a processor $p_k$).

Let $D \subseteq C_{V_s}$ be the subset of the orderings for $G_s$ restricted such that if $o \in D$, then

$$\forall v_1, v_2 \in V_s, C_P(v_1) = C_P(v_2) \Rightarrow o(v_1) \neq o(v_2)$$
$D$ is the restriction of the synchronous orderings on $V_s$ to the hardware partitioning, where two vertices cannot have the same order if they share a resource. Note that if the design is not schedulable, $D$ will not exist.

Finally, consider the schedule. Let $S : G_s \rightarrow \mathbb{R}^{|V_s|+|E_s|}$ represent start times for all elements of the dataflow graph $G_s$.

Let $D' \subseteq D$ satisfy $(\forall o' \in D')$:

$$\forall v_1, v_2 \in V_s, e = (v_1, v_2) \in E_s \land C_P(v_1) \neq C_P(v_2)$$

$$\Rightarrow S(v_2) > S(v_1) + \text{exec}(v_1) + \text{exec}(e) \land o'(v_2) > o'(v_1)$$

If two components have a dependency through a remote message, then their start times are constrained by the start time of $v_1$, the duration of $v_1$, and the duration of the message represented by the edge $e$. This models the logical execution time semantics of time-triggered execution. Note that the addition of the edge time may push the order values farther apart by allowing other tasks to execute during the data transfer time, so the reduction involved in $D'$ may be significant. Again, we assume that the model is schedulable.

Since the final set of orderings $D'$ was constructed by reduction from the initial set of orderings $C_V$, any scheduling policy for ESMoL that enforces the constraints and partitionings shown above will maintain the synchronous semantics of the original Simulink model if the ESMoL model is schedulable. Note that we have not dealt with delays. The scheduling tool described in Porter et al[70] conforms to the constraints as described, if combined with the Stage1 logic to create local dependencies for transitive remote connections as described above. Unfortunately, the scheduler does not enforce end-to-end latencies well, an issue addressed conceptually in Chapter V.
Evaluation

Our approach for creating high-confidence designs varies somewhat from the traditional V-diagram development model (see Fig. 26). In the traditional model we move down the V, refining designs as we proceed, with the level of integration increasing as the project progresses. We recognize that system integration is often the most costly and difficult part of development. Lessons learned during integration frequently occur too late to benefit project decision-making. We aim to automate much of the integration work, and therefore shorten design cycles. Beyond that, we want to enable feedback of models and analysis results from later design stages back to earlier design cycles (along the dashed lines in the conceptual diagram) to facilitate rapid rework if necessary. The goal is that the overall project can rapidly move towards a correct implementation that most accurately reflects our current understanding of the design problem.

Our case study covers the incremental development of software for the Starmac quadrotor aircraft [76, 77]. We deployed our software to the same hardware as the Starmac controller (with the exception of our internal $I^2C$ link, where the Starmac design used a UART), and tested in
a hardware-in-the-loop environment which simulated the Starmac dynamics. Specifically, we conducted three development phases (each with a corresponding set of design models), each of which successively refined the design while preserving the component structure:

1. **Communications Test:** We designed and deployed a shell of the controller architecture, where the software controller components received and sent messages of the proper size, but the system functions only copied data from the input ports to the output ports of each component. The Mathworks xPC Target Hardware-in-the-Loop (HIL) simulator injected known data patterns into the deployed dataflow implementation to ensure that all data paths were valid given the configured schedule.

2. **Quad Integrator Test:** We designed and deployed a simplified version of the quadrotor which acted only along a single axis of motion, removing the rotational dynamics. We were able to validate our control design approach (see [84]), and determine a method for gain adjustments required for stable operation of the deployed controller.

3. **Quadrotor Test:** The final phase evaluated the full quadrotor dynamics and controller implementation. We tested trajectory tracking with the full platform delay effects.

Each of the three development phases answers a set of questions regarding the correctness of the design under nominal operating conditions:

- **Communications Test:**

  1. Is the hardware configuration valid for this software configuration?
  2. Does our deployment mapping communicate the right amounts of data round trip?
  3. Does the configured schedule avoid communication conflicts?
  4. Is data corrupted by the communication protocols or software?
  5. How much delay is introduced by the configured schedule?

- **Quad Integrator Test:**
1. Does our methodology for selecting stabilizing gains for the control loops adequately handle the schedule delay introduced by data buffering, network communication, and the calculated schedule?

2. Is our sampling process sufficient for the platform and essential control architecture?

3. Are there any numerical problems that arise in our functional dataflow implementation considering normal input value ranges?

- **Quadrotor Test:**

  1. Given the additional functions and dimensions in the dataflow, can we still properly answer all of the questions from the previous phase?

  2. Does the full configuration track a reference trajectory?

Fig. 27 is a conceptual depiction of our evaluation environment. The Mathworks xPC Target simulation software runs on a generic small-form-factor PC, with ethernet for configuration and data collection. The xPC system contains an 8-port RS-232 serial expansion card, which communicates with the controller hardware on one port. The simulator and controller send sensor and actuator data back and forth on a single full-duplex serial link running at 57600 baud. The controller hardware consists of two processor boards—the Gumstix Linux board runs the *OuterLoop* controller and the *RefHandler* data input tasks. The Gumstix board has access to an ethernet port, through which the host machine sends new controller software for both control boards. We also use secure shell connections to start and stop the controller, and to monitor for error messages which are printed to the console. An internal *I*²*C connection allows the two control boards to exchange sensor data and attitude control commands. The Robostix AVR board runs the *InnerLoop* attitude controller and the *DataHandler* sensor data distribution component. One Robostix UART device connects to the xPC simulator as described above. Digital I/O pins allow the monitoring of timing information for the Robostix. We embedded commands to toggle the I/O pins in the controller software, and connected the pins to the LogicPort logic probe. The probe software shows timing traces for evaluating schedule operation (as in Fig. 40). A software AVR simulator was also used to evaluate timing and stack usage for the software running on the AVR. The Robostix board runs FreeRTOS. A Windows virtual
machine on the host PC runs the ESMoL modeling tools, logic probe display software, and Simulink which configures and compiles models for the xPC target software. The Linux-based host itself runs the cross-compilers for the controller targets, and secure shell connections to the Gumstix board for status monitoring.

**Communications Test**

Fig. 28 displays the simple model used to test data flow over the communication channels. The blocks contain only pass-through elements – multiplexers, demultiplexers, and gains. With this model we verified that data flowed correctly through all of the data paths in the system. The
InnerLoop, OuterLoop, and DataHandler components were all realized in software from an ESMoL model, and deployed to the hardware platform. The execution of the components is controlled by a simple time-triggered virtual machine that releases tasks and messages at pre-calculated time instants.

The Mathworks xPC Target simulated the plant dynamics for this test, which in this case amounted only to signal generators to create known data for the simplified controller blocks, and scopes to visualize data received from the controller board. We compared the input and output traces for (delayed) equality (Fig. 29).

During this phase we found problems with the $I^2C$ communications link. The scheduling and timed execution both required precise coordination to prevent data corruption. We also manually discovered a deadlock condition in our communications controller logic. Increasing the speed of the $I^2C$ link from 100 kbits/sec to 400 kbits/sec resolved both the scheduling problem and the deadlock.

Quad Integrator Model

Our second evaluation phase controls a continuous-time system whose model represents a simplified version of the quadrotor UAV. This model still follows the basic component architecture for the control design (see Fig. 8), but excludes the nonlinear rotational dynamics of the full quadrotor while retaining the difficult coupled stability characteristics. Fig. 31 shows a Simulink model...
Figure 30: Simulink model of a simplified version of the quadrotor architecture.

Figure 31: Simplified quadrotor plant dynamics. The signal lines leading off the picture are signal taps used for online stability analysis.

containing the simplified dynamics. The example model controls a stack of four integrators (and motor lag) using two nested PD control loops, as shown in the Simulink diagram of Fig. 30. The Plant block contains the integrator models representing the vehicle dynamics. The two control loops (InnerLoop and OuterLoop, as shown in Fig. 30) are deployed to the Robostix and Gumstix processors, respectively. We refer to this example as the Quad Integrator model. All of the controller components run at a frequency of 50Hz.

Our controller evaluation method is based on sector theory, proposed originally by Zames[62] to analyze nonlinear elements in a control design. Sectors provide two real-valued parameters which represent bounds on the possible input/output behaviors of a control loop. Kottenstette presented a sector analysis block for validating a control design in Simulink[65]. We propose to use the same
structure to verify the deployed quadrotor control software online. This method is described more fully in Porter et al[84]. A few concepts make this approach appealing for our case:

1. For a given component, the sector measures behavior simultaneously over multiple inputs and outputs, so only one sector analyzer is required per control loop.

2. Our passive abstraction of the system design (described below) allowed us to use a sector analyzer for each control loop to quickly isolate problem components in the deployed design.

Passive control requires that controllers use energy received from inputs or stored previously, introducing no new energy into the environment[85]. If the plant dynamics were passive, we would have considerable freedom in setting gains and choosing control structures. The zero-order hold outputs can introduce small amounts of new energy to the environment during rapid velocity changes, so each of the control loops must mitigate small amounts of “active” behavior. The sector bound $a$ quantifies the energy-generating behavior of each control loop. In our quadrotor system, we expect the bound $a$ to be small and negative and choose the gains appropriately. The result from Kottenstette indicates that the condition $k < -1/a$ is sufficient to ensure stability in these situations (where $k$ is the configured gain of the control loop)[65].

![Conceptual nested loop structure of the controller.](image)

This particular design must be evaluated from the innermost loop to the outermost loop in order to make sense of the gain constraints. Fig. 32 shows the nested loop structure of the design. The actual design and implementation are complicated by the physical architecture of the digital realization:

1. Sensors acquire digital attitude and position information only, so velocities must be estimated.
2. The controller components are deployed to different processors in the digital implementation, as described previously. Components on the two processors exchange data messages using a time-triggered protocol.

3. Motor thrust commands are issued periodically using a zero-order hold. As discussed previously the hold introduces additional energy back into the environment, violating the passivity condition.

The sector blocks are attached around each controller, so input and output ports are oriented from the point of view of the control element. The output of the controller (input to the rest of the system) is connected to the sector analyzer input port. The signal controlled by the controller (before the error term is formed) is part of the input to the controller, but from our point of view it is the output of the system, so it connects to the sector analyzer output port. Fig. 33 displays the connection of the sector search block around the position control gain for our example. $K_x$ is the proportional gain for the outer loop PD controller, and $K_v$ is the derivative gain.

Figure 33: Sector analysis block (SectorSearch) connection around the position controller.

For this test we selected a square wave reference input near the highest frequency admissible by the controller. Platform effects caused a significant deviation from our ideal sector estimates and bounds, as illustrated by the sector bound changes in Table 13. Fig. 34 illustrates the evolution of
Figure 34: Sector value evolution over time for the quad integrator.

<table>
<thead>
<tr>
<th>Signal</th>
<th>Original Bound</th>
<th>Simulated Sector</th>
<th>Measured Sector</th>
<th>Delta</th>
<th>New Bound</th>
<th>New Sector</th>
</tr>
</thead>
<tbody>
<tr>
<td>Angular Velocity</td>
<td>-1.333</td>
<td>-1.2292</td>
<td>-1.2963</td>
<td>-0.0671</td>
<td>-2.667</td>
<td>-1.4568</td>
</tr>
<tr>
<td>Angle</td>
<td>-0.5</td>
<td>-0.0295</td>
<td>-0.4831</td>
<td>-0.4536</td>
<td>-1.0</td>
<td>-0.0068</td>
</tr>
<tr>
<td>Velocity</td>
<td>-0.5</td>
<td>-0.1856</td>
<td>-0.4830</td>
<td>-0.2974</td>
<td>-1.0</td>
<td>-0.9324</td>
</tr>
</tbody>
</table>

Table 13: Sector value comparisons for simulation and execution on the actual platform.

the collected sector data over time. For each digital control signal the table records the following (by column):

1. Original Bound: the sector bound based on the original gain value ($-\frac{1}{k}$).

2. Simulated Sector: the sector value recorded in simulation.

3. Measured Sector: the initial sector value measured on the platform.

4. Delta: the sector difference between the measured and simulated values.

5. New Bound: the sector bound based on the newly adjusted gains.

6. New Sector: the sector value measured on the platform with the new gains.

Although the initial platform gains satisfied the sector stability conditions analytically and in simulation (comparing the Bound column to the Simulated column in the table), the overall system response when deployed to the target platform resulted in significant position overshoot. The measured sector value for position measured the farthest from the predicted value, and exceeded the gain bound for stability ($-1/k$), though no evidence of instability was visible in the plot of the
output trajectory. As all of the gains moved right up to the edge of their bounds when deployed, we reduced all of the gains by \( \frac{1}{2} \). Note that changing the gains changes the acceptable sector bound as well as the actual sector bounds themselves (as shown in Table 13). After adjusting the gains all of the sector values fell within the bounds.

On closer inspection we discovered that the most significant platform effect was a non-ideal position gain condition for signals with frequencies too close to the sampling rate. Fig. 35 shows a comparison of the ideal frequency response of the outer loop controller block with an empirically measured frequency response for the same controller block deployed on the target hardware. Note the spike at the right-hand side of the plot in Fig. 35(b). This is a nonlinear gain anomaly due to the effects of the saturation block, and which appears only for signals with frequencies right near the Nyquist sampling rate. The remedy was to add a simple input filter to cut off frequencies too close to the sampling rate. This effectively slows down the possible commands that can be issued to the system. The sector analysis blocks helped identify the position control component as the element whose behavior was farthest from predicted when deployed to the platform. Adding a rate limiter block to the reference input resolved the problem. Note that the full quadrotor model already included a similar (but more complex) rate limiter.

![Image](image_url)

Figure 35: Magnitude frequency responses for the quad integrator.

The Quad Integrator model simulation exposed a few interesting and unanticipated defects in our design, beyond the gain anomaly detected by the sector analysis. The most significant problem was the asynchronous arrival of the input sensor data. Since the input data transfers were not
synchronized with the controller schedule, we had to add a double-buffer to the UART data handler in order to eliminate data corruption.

**Quadrotor Model**

The final development phase integrated the full dynamics of the quadrotor, comprised of the full data paths and nonlinear functions of the controllers. Figs. 36 - 38 show details from the full Simulink model for the quadrotor. In the top-level design model (Fig. 36), the `robo_stix` block (Fig. 37) contains the functional specifications for the `DataHandler` (`sensor_convert` block) and the `InnerLoop` (`inner_loop` block, also Fig. 38) software components. Likewise the `gum_stix` block and the `ref_data` block specify functions for the `OuterLoop` and `RefHandler` software components.

Figure 36: Simulink model of the Starmac quadrotor helicopter.

We used the LogicPort probe to assess the correctness of the schedule. The configured schedule (Fig. 39) correlates with the schedule points measured by the LogicPort analyzer for the tasks and messages on the Robostix board (Fig. 40). Our experimental configuration did not provide a similar means for accurate measurement of the timing on the Gumstix board, though we can observe that message transfers start and end as predicted when task interference is absent. Task interference
was only observed for misconfigured schedules, or when other non-controller Gumstix processes created heavy loads, delaying the controller. Both schedule-based and load-based interference were eliminated for nominal operation. Fig. 41 illustrates tracking behavior for the xPC-simulated Quadrotor, where the real-time controller implementation runs on the actual controller hardware. The dashed curves represent the commanded $x$, $y$, and $z$ positions as shown, and the solid lines show the actual trajectory achieved by the HIL simulated helicopter using the deployed controller code.

Our first move to the full quadrotor model uncovered numerical problems with some of the emulated floating-point functions provided by the gcc ARM cross-compiler. This forced us to implement our own versions of the single-precision absolute value, signum, and minimum functions for the OuterLoop component. This problem was new to this phase of the evaluation because the rate limiter was not present in the Quad Integrator model.

**Lessons and Future Work**

Probably the greatest difficulty in our work has been dealing with the large number of moving parts involved in the development of the modeling language and tools, the modeling and implementation
of design examples, and the configuration of the development tools and execution environment for the target platform. Our MIC-based solution only covered a part of the entire problem. We only lightly addressed target system configuration, automated updates of the ESMoL model to track changes in the Simulink design, and runtime assessment (of both the simulator for plant dynamics and the target platform with the deployed code). We developed a technique for runtime assessment of controller stability as covered in Porter et al[84] (described partially in the Quad Integrator evaluation section), but it was difficult to automate due to the limited number of free data paths available for debugging in our chosen target system. The integration of third party libraries in the development of our tools, and variations in platform module behavior were not directly addressed by our techniques, though they consumed significant development and testing time.

The next frontier in ESMoL development should be control loop modeling and analysis. Control design formalisms abound, each with its own particular features and capabilities. Passivity and the more general sector analysis formalisms are good examples of compositional frameworks which could be encoded in modeling tools[86] and which could support incremental development.
Figure 39: Schedule configuration for the quadrotor.

Figure 40: Timing diagram for the Robostix AVR running the inner loop controller.
Figure 41: Trajectory tracking for the quadrotor implementation.
CHAPTER IV

INCREMENTAL SYNTAX ANALYSIS FOR COMPOSITIONAL MODELS:

ANALYZING CYCLES IN ESMoL

The ESMoL language is built on a platform which provides inherent correctness properties for well-formed models. The properties include functional determinism, deadlock freedom, and timing determinism. Establishing well-formedness for a particular model sometimes requires sophisticated syntactic analysis of the global model structure. Where possible, for particular decomposable syntactic analysis problems we would like to use the model structure to improve efficiency of analysis. As design changes are made to the model, we would like to limit analysis to those components that are affected by the modification.

Overview

Syntactic Analysis Challenges

One particular analysis problem concerns synchronous execution environments and system assembly. In dataflow models of computation we are often concerned with so-called “algebraic” or delay-free processing loops in a design. Many synchronous formalisms require the absence of delay-free loops in order to guarantee deadlock freedom [87] or timing determinism [8]. This condition can be encoded structurally into dataflow modeling languages – for example Simulink [88] analyzes for algebraic loops and attempts to resolve them analytically. In the Ptolemy dataflow design environment, such causality loops complicate scheduling requiring fixed-point iteration to ensure convergence of results[89]. In this work we only consider the structural problem of loop detection in model-based distributed embedded system designs.

Cycle Detection in ESMoL Models

We propose a simple incremental cycle detection technique with the following characteristics:
• The algorithm uses Johnson’s simple cycle enumeration algorithm as its core engine[90]. Johnson’s algorithm is known to be efficient[91]. We use cycle enumeration rather than simple detection in order to provide useful feedback to the designers.

• The algorithm exploits the component structure of hierarchical dataflow models to allow the cycle enumeration to scale up to larger models.

• A simple incremental interface is created and stored in each component as the analysis processes the model hierarchy from the bottom up. The method should scale to large designs without imposing onerous data storage requirements on the model, and allow highly efficient recomputation of the cycle analysis when design changes occur in the model.

• The technique will not produce false positive cycle reports, though it may compress multiple cycles into a single cycle through the abstraction. Fortunately, full cycles can be recovered from the abstract cycles through application of the enumeration algorithm on a much smaller graph.

Zhou and Lee presented an algebraic formalism for detecting causality cycles in dataflow graphs, identifying particular ports that participate in a cycle. [52]. Our work traverses the entire model and extracts all elementary cycles, reporting all ports and subsystems involved in the cycle. Our approach is also inspired by work from Tripakis et al, which creates a richer incremental interface for components to capture execution granularity as well as potential deadlock information[2]. Their approach is lossless, in that it retains sufficient detail to faithfully represent dataflow structure and execution granularity. It is much more complex in both model space and computation than our approach. Our formalism does not aim to pull semantic information forward into the interface beyond connectivity. In that sense our approach is more general, as it could be applied to multiple model analysis problems in the embedded systems design domain.

The KPASSA model analysis tool described by Boucaron et al [92] performs task graph scheduling analysis for latency-insensitive synchronous designs. Their formal model leans heavily on loop structures, and as such one component of their tool relies on an implementation of Johnson’s cycle
enumeration algorithm[93]. Their formal model is specific to a particular model of computation, and their application of cycle checking is only one small component of that solution.

**Tools and Techniques**

**ESMoL Component Model**

As the ESMoL language structure is documented elsewhere, we only cover details relevant to incremental cycle checking. ESMoL is a graphical modeling language which allows designers to use Simulink diagrams as synchronous software function specifications (where the execution of each block is equivalent to a single bounded-time blocking C language call). These specifications are used to create ESMoL component type blocks. ESMoL components have message structures as interfaces, and the type specification includes a map between Simulink signal ports and the fields of the input and output message structures.

Once software component types and interfaces have been specified, ESMoL designers instantiate those components into a distributed deployment model. ESMoL allows the separate specification of the logical data flow, the mapping of component instances to hardware, timing information for tasks executing those components, and timing for messages sent over a time-triggered communication bus. Code generated from the models conforms to an API for time-triggered execution. A portable virtual machine implementation of the API allows execution in simulation, hardware-in-the-loop, and fully deployed configurations[69].

ESMoL deliberately provides an unusual degree of freedom in creating software component types. A designer can include Simulink references from any part of an imported dataflow model, and instantiate them any number of times within the type definitions. The partition of functions into ESMoL types allows the designer to control the granularity of functions assigned to distributed tasks. Tasks can distribute functions over a time-triggered network for performance, or replicate similar functions for fault mitigation. This level of flexibility requires automatic type-checking to ensure compatibility for chosen configurations. Beyond interface type-checking, structural well-formedness problems arise during assembly such as zero-delay cycles. Model analysis must ensure well-formedness.
Cycle Enumeration

To implement cycle enumeration we use the algorithm Johnson proposed as an extension of Tiernan’s algorithm [94] for enumerating elementary cycles in a directed graph[90]. Both approaches rely on depth-first search with backtracking, but Johnson’s method marks vertices on elementary paths already considered to eliminate fruitless searching, unmarking them only when a cycle is found. Johnson’s algorithm is polynomial ($O((n + e)c)$, where $n$, $e$, and $c$ are the sizes of the vertex, edge, and cycle set, respectively), and is still considered the best available general cycle enumeration method[91]. We created an implementation of Johnson’s algorithm in C++ using the Boost Graph library[95].

Hierarchical Graphs

For formally describing our incremental approach we use the algebra of hierarchical graphs introduced by Bruni et al[96]. We repeat here their first definition: a design is a term of sort $D$ generated by

$$D ::= L_{\bar{x}}[G]$$

$$G ::= 0 \mid x \mid l < \bar{x} > \mid G \parallel G \mid (\nu \bar{x})G \mid D < \bar{x} >$$

Here term $G$ represents a hierarchical directed graph, $D$ is an edge-encapsulated hierarchical graph, $x$ is a vertex, $\bar{x}$ is a list of vertices in $G$ (for which $|\bar{x}|$ is the corresponding set), $l \in E$ (edge labels of $G$, where edges can have n-ary connectivity ), $L_{\bar{x}} \in D$ ($D$ are the design labels of $G$ and $\bar{x}$ are interface vertices in $L$), $G \parallel G$ is parallel graph composition which merges vertices with common names, $(\nu \bar{x})G$ restricts the interface of graph $G$ to exclude vertices in $|\bar{x}|$, and the notation $D < \bar{x} >$ maps the vertices from the interface of $D$ to the vertices listed in $\bar{x}$ (renaming vertices internal to the design for the external interface). Finally $[G]$ indicates the graph corresponding to the term $G$.

Note that the term design is used for components in the hierarchy, each with a type and a set of interface vertices. Unfortunately in the realm of graph theory the term component has a different
meaning. This algebraic model was conceived to more easily compute structural equivalence between hierarchical graphs. Bruni et al prove that syntactic equivalence between two design models expressed as term algebras corresponds to isomorphism in their respective graphs\cite{96} and consequently equivalent behaviors in computational formalisms mapped to the algebras. Their formalism also includes a definition of \textit{well-typedness}, where types defined on the vertex set are only connected if their types are compatible. Finally they define \textit{well-formedness} for hierarchical graphs which includes \textit{well-typedness} as a condition. We do not define the entire formalism here, only enough to understand the essence of the connections between the terms and the graphs that they represent.

\section*{Incremental Cycle Analysis}

Our intention is to support a design and analysis work flow that includes incremental analysis steps. For example, a design may analyze part of the design before integrating it into a larger part of the system. In our work flow, we envision storing the results of that first analysis along with some interface data to reduce the cost of the second analysis. The same should hold true for the system design. We should be able to analyze the system design efficiently, calculating incremental analysis interfaces. When the system models are revised, whether by adding, removing, or modifying components we can isolate the effects of the change on the cost of the analysis. Cycle analysis is a useful example, but our aim is to tackle this problem more generally.

\section*{Formal Model}

Let $G$ be a well-formed hierarchical graph (as in Bruni\cite{96}). To get more comfortable with the notation, first note that graph $G$ itself (without hierarchical structure) can be given as:

\begin{equation}
G = (\| x \| (\| (u,v) \in E \| l < u, v >))
\end{equation}

which is the parallel composition of the individual edge graphs of $G$, merged at their common vertices.

Let $C(G)$ be the set of elementary cycles in $G$, and let $P(G, u, v)$ be the subgraph of $G$ containing all of the paths from vertex $u$ to vertex $v$. 

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Consider a design of type $W$. Let $W^p_\bar{x}[G]$ represent a parent design object in a graph hierarchy with interface vertices $\bar{x}$, and let $W^c_\bar{x}[G]$ be the design children of $W^p$ ($\|W^c_\bar{x}\| \subset \|W^p\|$). Then neglecting vertex hiding and renaming to simplify the illustration, we have the following:

$$W^p_\bar{x}[G] = W^p_\bar{x}\left[\|x_h\| \|\{(j,k) \mid l < j, k \}\| \|i_i\|W^c_\bar{x}[G]\|\right]$$

Eq. 12 describes the design $W^p$ in terms of its design children $W^c_\bar{x}$, internal vertices $x_h$, and edges $l < j, k$.

We introduce a new label $l_c$ into the sort for edges ($\mathcal{E}$), which is used to connect vertices at the boundaries of a design, abstracting the interface connectivity of the design. Introduce a new mapping $A : \mathcal{D} \rightarrow \mathcal{D}'$ from the designs of $G$ to designs in a new graph $G'$. $G'$ is identical to $G$, but adds the new edge label. This is the interface that we will use for incremental cycle analysis.

$$A(W^c_\bar{x}[G]) = W^c_\bar{x}\left[\|x_h\| \|\{(j,k) \mid l < j, k \}\| \|i_i\|W^c_\bar{x}[G]\|\right]$$

In this abstraction function the child designs are replaced by a much simpler connectivity graph.

We introduce two functions to support the algorithm:

$$R(A(W^c_\bar{x}[G])) = W^c_\bar{x}\left[\|x_{\bar{x}_i}\| \|\{j,j \mid l < j, k \}\| \|i_i\|R(A(W^c_\bar{x}[G]))\|\right]$$

$$S(W^p_\bar{x}[G]) = W^p_\bar{x}\left[\|x_h\| \|\{(j,k) \mid l < j, k \}\| \|i_i\|R(A(W^c_\bar{x}[G]))\|\right]$$

$R(\cdot)$ and $S(\cdot)$ map designs in $G$ to an abstracted design which only has connectivity edges for each child design. In other words, when analyzing a component of $G$ we use the incremental interface data for each child component rather than its full details. This is a useful abstraction for cycle detection: we can exploit the graph hierarchy to enumerate simple cycles more efficiently.
Algorithm Description

Assume we have a function \( \text{FINDALLCYCLES} : G \rightarrow 2^G \) which enumerates all elementary cycles in a graph \( G \), returning sets of subgraphs. Then Algorithm 1 adapts the general algorithm \( \text{FINDALLCYCLES} \) to the hierarchical graph structure described above. We assume that \( G \) has a unique root design, and that we have a function \( \text{modified} : D \rightarrow \text{boolean} \) which indicates whether a particular hierarchical component has been modified since the last run. New components in the model are considered modified by default.

Algorithm 1 Hierarchical cycle detection

1: \( \text{cycles} \leftarrow [] \)
2: \( \text{ifaces} \leftarrow \{ \} \)
3: \( \textbf{function} \ \text{FINDHCYCLES} ( [[W_p \bar{x}]_G]) \)
4: \( \textbf{for all} \ W_c i \bar{x}_i_G \in W_p \bar{x}_G \) \( \textbf{do} \)
5: \( \text{FINDHCYCLES}([[W_c_i \bar{x}_i_G]]) \)
6: \( \textbf{end for} \)
7: \( \text{modified}(W_p \bar{x}_G) \leftarrow (\text{modified}(W_p \bar{x}_G)) \lor (\forall i, \text{modified}(W_c_i \bar{x}_i_G)) \)
8: \( \textbf{if} \ \text{modified}(W_p \bar{x}_G) \) \( \textbf{then} \)
9: \( T \leftarrow S(W_p \bar{x}_G) \)
10: \( \text{cycles} \leftarrow [\text{cycles}; \text{FINDALLCYCLES}(T)] \)
11: \( \text{ifaces}[p] \leftarrow A(T) \)
12: \( \textbf{end if} \)
13: \( \textbf{end function} \)
14: \( \text{FINDHCYCLES}(G) \)

The algorithm performs a depth-first search on a hierarchical graph. If the component has been modified, we compute connectivity interfaces for each subcomponent and check for cycles in the parent component – the connectivity graph interface is substituted for each subcomponent. The modification status is propagated up the hierarchy as the algorithm progresses. Each component which has a modified child will also be marked as modified. The cycles are accumulated as the algorithm ascends to the top of the model.

The runtime for the extended algorithm is slightly worse than Johnson’s algorithm in the worst case, as it must also compute the interface graphs. In the average case the cycle checking proceeds on graphs much smaller than the global graph, offsetting the cost of finding paths in each subgraph.

Further, if the incremental interface edges are stored in the model following the analysis, then
scalability is enhanced when incrementally adding functions to a design. Cycle analysis is then restricted to the size of the new components together with the stored interfaces.

**ESMoL Language Mapping**

Now to map ESMoL logical architecture models onto this cycle-checking formal model we use the following rules:

\[
\text{Subsys} ::= L^{\text{Subsys}}_{\text{i},\bar{\text{o}}} \parallel \text{Dataflow}
\]

\[
\text{Dataflow} ::= 0 \mid x \mid l_D < \bar{x} > \mid \text{Subsys} < \bar{x}, \bar{x} >
\]

\[\mid \text{Dataflow} \parallel \text{Dataflow} \mid (\nu \bar{x}) \text{Dataflow}\]

\[
\text{MsgType} ::= M_{\nu,\text{ext}}
\]

\[
\text{SysTypeDef} ::= \text{Subsys} < \bar{\text{i}}, \bar{\text{o}} > \mid l_S < x, x >
\]

\[\mid \text{SysTypeDef} \parallel \text{SysTypeDef} \mid \text{MsgType} < \bar{x}, y >\]

\[
\text{SysType} ::= L^{\text{Sys}}_{\bar{\text{y}},\bar{\text{e}}} \parallel (\nu \bar{x})(\nu \bar{e}) \text{SysTypeDef}
\]

\[
\text{LogicalModel} ::= \text{SysType} < \bar{\text{i}}, \bar{\text{o}} > \mid l_L < o, i >
\]

\[\mid \text{LogicalModel} \parallel \text{LogicalModel}\]

Briefly (from the bottom rule to the top), logical models consist of component blocks (**SysType**) whose interface ports connected by edges. Component blocks are specified by Simulink dataflow blocks (**Dataflow**) whose interface ports are connected either to other Simulink dataflow blocks or to fields in message instances. Each message instance (**MsgType**) inside a system component type block also has an interface vertex (**y**) which faces outward, and all other vertices are hidden within the component \((\nu \bar{x})(\nu \bar{y})\text{SysTypeDef}\). At the logical architecture model level, data is exchanged via messages which aggregate the individual dataflow connections within the components. **Dataflow** blocks are built up from connections between functional vertices and between the interfaces on composite subsystem blocks (**Subsys**). These each correspond to sorts in the ESMoL term algebra.
Let $i$, $o$, and $e$ be vertex sorts corresponding to input ports, output ports, and message elements respectively. Let $s$, $c$, $f$, and $d$ be edge sorts (of $l_D$, above) representing signal edges, connectivity edges (as described above to support the incremental interface), $f$ for Simulink primitive function blocks, and $d$ for delay blocks. The $f$ function edge sorts are n-ary, so each function block can have an arbitrary but finite number of input and output connections. For $l_S$ define the sorts (given with their interfaces) $l^{b,b} < o, i >$, $l^{m,b} < e, i >$, and $l^{b,m} < o, e >$. These represent the three different connection types in a $\text{SysType}$ specification, for connecting between ports of Simulink blocks (from outputs to inputs) ($l^{b,b}$), from message elements to Simulink input ports ($l^{m,b}$), and from Simulink output ports to message elements ($l^{b,m}$).

Finally we give an encoding of terms representing ESMoL models into the more general hierarchical graph algebra:

\[
\begin{align*}
\bar{x} &= x \\
L_{\text{Subsys}}^{\text{Dataflow}}[\bar{x}, \bar{x}] &= L_{\bar{x}}[G][\bar{x}] \\
M_{\text{SysDef}}[\bar{x}, x, y] &= L_{\bar{x}}[G][\bar{x}] \\
L_{\text{SysDef}}^{\text{Dataflow}}[(\nu x)(\nu e)\text{SysTypeDef}][\bar{x}] &= L_{\bar{y}}[G][\bar{x}] \\
l_D[\bar{x}] &= l[\bar{x}] \\
l_s[\bar{x}, x] &= l[\bar{x}] \\
(\nu \bar{x})\text{Dataflow} &= (\nu \bar{x})G
\end{align*}
\]

(17)

The encoding assigns the various layers of hierarchy from the ESMoL component type system to hierarchical designs in the graph. Edges from all layers map to (possibly generalized) edges in the new graph, and ports map to vertices.

The final piece is the application to finding delay-free loops. For a given ESMoL model, simply remove all delay edges (sort elements $d$). Then invoke the algorithm. For the results, if a cycle is found in a component we can construct a more detailed cycle model by substituting paths from the connectivity edge sort with their more detailed equivalents in the descendants of the component.
(recursively descending downwards until we run out of cycle elements). Call this subgraph the *expanded cycle*. Repeating the cycle enumeration algorithm on these structures should yield the full set of elementary cycles, and still retain considerable efficiency as we are only analyzing cycles with possible subcycles, which can be a relatively small slice of the design graph.

**Evaluation**

**Fixed-Wing Aircraft Example**

Our case study covers cycle analysis of the control design for a fixed-wing aircraft. The Simulink model (Fig. 42) shows the four controller blocks and the sensor data handler. The particulars of the control architecture are not important for this example, but Kottenstette covers them in detail[97]. The controller has five software functions which are specified as Simulink model blocks, and a dynamics component (the Cessna plant block). The **MDL2MGA** model importer creates a structural replica of the Simulink model in the ESMoL modeling language. We use subsystems from the replica to specify the function of synchronous software components. Fig. 43 illustrates one possible configuration of the fixed wing controller components. In this particular configuration (Fig. 43) the entire dataflow is included in one type definition, which means that the entire system

![Simulink Fixed Wing Controller Model](image-url)
Figure 43: Synchronous data flow for Fixed Wing Controller

will execute together as a single synchronous function with all blocks firing at the same rate. This particular configuration is useful for illustration, but is not the most practical implementation choice.

**Incremental Analysis Results**

Table 14 contains data from the analysis of the fixed wing model. The first pass was performed incrementally, with each subcomponent of the top level model analyzed first. Then the top level is analyzed using the stored path edges in the lower models. The table reports two run times for the analysis of each component – the first is the processing time required to find the abstract cycles only, and the second is the full analysis which finds the expanded cycle for each abstract cycle (enumerating possibly multiple cycles per abstract cycle). The table also displays the number of hierarchical components visited and the number of individual model elements visited, together with the number of abstract cycles found and the total number of cycles. The table row labeled *top level (incremental)* contains the results for the analysis of the top level of the model once the individual path interfaces had been created for each of its subcomponents. The second pass (labeled *top level (full)*) analyzed the entire fixed wing model at once, reporting the same quantities. Our assessment of the scalability of the approach is inconclusive for three reasons – 1) the model size is
moderate, so overhead is likely large enough to be a significant factor in all of the run times, 2) we
would need a comparison with time taken to process a fully flattened model, including the flattening
traversals, and 3) we need to find larger models for our test set. The analyzer found 18 abstract
cycles and 54 detailed cycles at the top level for both passes. The velocity\_controller component also
contained a single abstract cycle (consisting of two detailed cycles). Note that we analyzed for all
cycles rather than only delay-free cycles to assess scalability. Total runtime was roughly equivalent
between the full and incremental methods for this particular model. The results so far are promising
but inconclusive as far as improved performance.

<table>
<thead>
<tr>
<th>Component</th>
<th>Abstract Run Time (s)</th>
<th>Abstract Cycles Found</th>
<th>Top level (incremental)</th>
<th>Top level (full)</th>
</tr>
</thead>
<tbody>
<tr>
<td>alpha_beta_mu_controller</td>
<td>0.9</td>
<td>0</td>
<td>2.3</td>
<td>7.9</td>
</tr>
<tr>
<td>gamma_chi_controller</td>
<td>1.6</td>
<td>0</td>
<td>1.6</td>
<td>55.1</td>
</tr>
<tr>
<td>gamma_chi_mu_sensor</td>
<td>1.3</td>
<td>0</td>
<td>1.3</td>
<td>55.1</td>
</tr>
<tr>
<td>omega_controller</td>
<td>0.9</td>
<td>0</td>
<td>0.9</td>
<td>0.9</td>
</tr>
<tr>
<td>velocity_controller</td>
<td>0.6</td>
<td>0</td>
<td>0.6</td>
<td>0.8</td>
</tr>
<tr>
<td>Totals</td>
<td>7.6</td>
<td>19</td>
<td>60.6</td>
<td>60.5</td>
</tr>
</tbody>
</table>

Table 14: Cycle analysis comparisons for the fixed wing model.

Figs. 44 and 45 display a subset of the velocity\_controller component which contains a cycle,
along with the expanded cycle for the component, in order to illustrate the cycle refinement in
greater detail. The abstract cycle search discovered the presence of a cycle within the component,
but part of the cycle lies within a subcomponent (anti\_windup\_control). The cycle detection for
anti\_windup\_control created a single path edge in the interface between the In1 port and the Out2
port, which corresponds to two paths within anti\_windup\_control. The full cycle as shown (Fig. 45)
is constructed in the analyzer, and then one more pass of Johnson’s algorithm resolves the two cycles
within the full cycle graph as reported in Tab. 14. The extracted cycle graph is much smaller (13
elements) than the corresponding fully flattened velocity\_controller model, which would contain 60
elements.
Figure 44: Detail of the components involved in the cycle found in the velocity controller.

**Future Work**

The current implementation is integrated into the ESMoL tool suite for the Generic Modeling Environment[71], but thorough scalability testing requires larger models.

One interesting observation is the generality of the approach. Algorithm 1 very nearly captures a generic procedure for bottom-up incremental syntactic analysis of hierarchical graphical models. Algorithm 2 proposes such a generic template. A complete study of such generic structural analysis techniques should include consideration of the effects of the component processing order on the accuracy of the result.
Algorithm 2 Hierarchical cycle detection

1: results ← []
2: ifaces ← {}
3: function analyze( [[W_p^i[G]]])
4:     for all W_c^i[G] ∈ W_p^i[G] do
5:         ANALYZE([[W_c^i[G]]])
6:     end for
7:     modified(W_p^i[G]) ← (modified(W_p^i[G]) ∨ (∨_c modified(W_c^i[G])))
8:     if modified(W_p^i[G]) then
9:         T ← ANALYZESTRUCTURE(W_p^i[G])
10:        results ← [results; COLLECTRESULTS(T)]
11:        ifaces[p] ← CREATEINTERFACE(T)
12:     end if
13: end function
14: ANALYZE(G)

Two immediate applications of this generic incremental method in ESMoL embedded control system designs are 1) automated sector analysis for passivity and/or stability and 2) quantization interval analysis for data precision and overflow. Both represent a static analysis of possible system behaviors that can be encoded syntactically. In both cases component interface data requirements are small, and computation is fairly efficient.
Figure 45: Full cycle for the velocity controller.
CHAPTER V

INCREMENTAL TASK GRAPH SCHEDULE CALCULATION

Analysis of semantic correctness properties depends on formal representations of the system behaviors represented by the model. The difficulty with semantic analysis is that concurrent behaviors over time may not correlate well with the structure of the model, since a proper behavior model will capture the numerous interactions between components and their effects on seemingly unrelated components. Semantic analysis is usually more computationally expensive than syntactic analyses, because of the need to look at states of the model over time. As discussed previously, incremental techniques may provide efficient analysis if they do not introduce behavioral approximations which are too conservative. The difficulty lies in finding useful decompositions of behavioral abstractions which will allow incremental analysis.

Overview

Semantic Analysis Challenges

A well-formed ESMoL model represents a particular set of system behaviors. Prior to schedule calculation the set of possible behaviors is underdetermined, leading to phenomenally large numbers of possible schedule configurations, many of which are essentially equivalent from the point of view of the requirements. ESMoL requires a scheduling technique which can use the timing and dependency information in the model to create a valid configuration for the execution of tasks and messages in the time-triggered network. Of particular concern is the satisfaction of end-to-end latency constraints for scheduled task graphs. Many scheduling techniques support offsets and local deadlines to support specified dependencies, but do not specify how those offsets and deadlines should be determined.

We need a scalable schedule calculation tool which supports iterative rework of design components and round-trip integration of the scheduling tool in the design flow.
Schedule Analysis and Calculation

We present here a conceptual design for an incremental scheduling and allocation algorithm which addresses the problems of scalability by incrementally computing task schedules for additional latency constraints. Specifically we propose a modification of the Bubble Scheduling and Allocation (BSA) algorithm [98] to handle node-locked tasks and incremental addition of new tasks, network data communication, and latency requirements.

Tools and Techniques

Task Graph Scheduling and Abstractions

Kwok and Ahmad present a detailed evaluation and comparison of many task graph scheduling techniques [33]. We will rely on their description of the task graph scheduling problem, and describe a few of the useful scheduling abstractions that are common to some of the algorithms. We are most interested in the Arbitrary Processor Network (APN) class of scheduling algorithms, which assign tasks to processors and start times to tasks in a particular network topology. A task graph is a directed acyclic graph (DAG) $G = (V, E)$, where vertices correspond to computational tasks having a known, bounded execution time, and edges correspond to messages transferred over a communication network. Vertices having no inbound edges are denoted entry vertices or sources, and vertices having no outbound edges are denoted exit vertices or sinks. Each vertex and edge has a set of weights, corresponding to the computation time required to execute the task represented by the node or to transfer the message represented by the edge. These execution times may differ for each processor or network link, so task and message weights are given as functions of vertex and processor (or edge and network). Let $w(v_i, P_k)$ be the task computation cost (execution time) for vertex $v_i$ on processor $P_k$. Likewise let $c(v_i, v_j, N_l)$ be the cost (transfer time) for the message from vertex $v_i$ to vertex $v_j$ over network link $N_l$. If two task vertices are scheduled on the same processor, then the communication cost of an edge between them is counted as zero. The vertex and edge weights may be determined by estimation, measurement, or static analysis. For schedule analysis they should be considered bounds on worst-case execution time rather than nominal values.
The \textit{b-level} (bottom-level) of a task vertex measures the longest path from the vertex to an exit vertex, counting both vertex execution time and data transfer times on the network. The \textit{t-level} (top level) measures the longest path distance from an entry vertex to the vertex in question, not including its own weight. These levels are frequently used in scheduling heuristics to assign priorities to tasks during scheduling. The \textit{t-level} roughly represents the earliest start time for a vertex in the DAG. A critical path (CP) is the longest path from a particular entry vertex to an exit vertex. In most of the literature the critical path is abstracted as the longest possible path in the task graph, bounding the \textit{b-level} values for that graph. For our purposes the task graphs may have multiple critical paths, where each corresponds to a latency requirement. More formally, given a task DAG $G$, latency requirements can be specified as triples $L_m = (v_i, v_o, t_m)$. Here $v_i$ indicates a source vertex, $v_o$ indicates a sink vertex, and $t_m$ indicates a time requirement within which all paths between $v_i$ and $v_o$ must execute. Then $CP_m$ represents the critical path from $v_i$ to $v_o$ for requirement $L_m$.

\textbf{Bubble Scheduling and Allocation (BSA)}

The Bubble Scheduling and Allocation (BSA) Algorithm\cite{98} first determines a critical path in the task graph by considering the \textit{b-level} and \textit{t-level} of task vertices. The CP determines a partition of the graph into IB (in-branch) vertices, CP (critical path) vertices, and NA vertices, which are successors to other kinds of vertices, originally called out-branch vertices in \cite{98}. We slightly adjust the b-level so that NA vertices have a value $blevel(v) = 0$, CP vertices start with $blevel(v_o) = 1$, and all predecessor b-level values are determined as described. Then the critical path and its in-branch vertices (IBVs) are scheduled serially to a single processor (the \textit{pivot processor}) using the routine \texttt{SERIALINJECT}. The function \texttt{SERIALINJECT} places all of the task graph vertices into descending b-level order. This is a slightly different presentation from the original description in Kwok\cite{98}, but it satisfies all of the precedence dependencies for each vertex in the critical path as required by the BSA algorithm. Our slightly modified b-level calculation also ensures that NA vertices are scheduled after latency-critical tasks.

The BSA algorithm (Algorithm 3) proceeds by iterating over the system processors (starting with the initial pivot processor) and testing scheduled vertices for migration. If a tested vertex
could start earlier on another processor, or if moving the vertex will reduce network data transfers without impacting the overall length of the critical path then the algorithm migrates the tested task.

We assume that all times are discretized, and thus given as integer values. Table 15 describes the critical functions, routines, and variables. Some of the algorithmic functions are not fully defined here to save space.

<table>
<thead>
<tr>
<th>Function or Routine</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>SERIALINJECT</td>
<td>Sequences the vertices in decreasing order of blevel, scheduling them to the specified processor.</td>
</tr>
<tr>
<td>SCHED</td>
<td>Starts from a migrated vertex, calculating start times for it and all successive vertices based on their processor assignment.</td>
</tr>
<tr>
<td>SORTPROCS</td>
<td>Orders the processors in the system according to a weighting function based on available space and cost-based proximity to the processors to which the start and end vertex are bound.</td>
</tr>
<tr>
<td>FINDCRITICALPATH</td>
<td>Calculates the \textit{b-levels} of the specified vertices with respect to the entry and exit, and marks the \textit{CP} and \textit{IB} vertices.</td>
</tr>
<tr>
<td>CALCSTART</td>
<td>Finds the best start time for a particular task vertex on the specified processor, subject to data dependencies.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>start</td>
<td>Array mapping vertices to their currently configured start times.</td>
</tr>
<tr>
<td>finish</td>
<td>Array mapping vertices to their currently configured end times.</td>
</tr>
<tr>
<td>dat</td>
<td>The earliest time at which a vertex could be scheduled, based on the arrival times of its data dependencies.</td>
</tr>
<tr>
<td>vip</td>
<td>For a given vertex, the immediate vertex predecessor whose data arrives last.</td>
</tr>
<tr>
<td>proc</td>
<td>Array mapping vertices to their currently assigned processor.</td>
</tr>
<tr>
<td>proclist</td>
<td>A priority-sorted list of processors.</td>
</tr>
<tr>
<td>pivot</td>
<td>The current processor.</td>
</tr>
</tbody>
</table>

Table 15: Function and variable definitions.
Algorithm 3 BSA Algorithm

1: \( \text{start} \leftarrow \{(n_0,0),(n_1,0),\ldots,(n_{\inf},0)\} \)
2: \( \text{finish} \leftarrow \{(n_0,0),(n_1,0),\ldots,(n_{\inf},0)\} \)
3: \( \text{dat} \leftarrow \{(n_0,0),(n_1,0),\ldots,(n_{\inf},0)\} \)
4: \( \text{vip} \leftarrow \{(n_0,n_0),(n_1,n_0),\ldots,(n_{\inf},n_0)\} \)
5: \( \text{function} \ BSA(G,H) \)
6: \( \text{proclist} \leftarrow \text{sortprocs}(H) \)
7: \( \text{pivot} \leftarrow \text{proclist}[1] \)
8: \( CP \leftarrow \text{findcriticalpath}(G) \)
9: \( \text{proc} \leftarrow \text{serialinject}(\text{pivot}, G, CP, H) \)
10: \( \text{for all} \ \text{idx} \in [1,\text{len(proclist)}] \text{ do} \)
11: \( \text{pivot} \leftarrow \text{proclist}[\text{idx}] \)
12: \( \text{for all} \ \text{n} \in \{\text{n}|\text{proc(n)} = \text{pivot}\} \text{ do} \)
13: \( \text{BestFT} \leftarrow 0 \)
14: \( \text{BestProc} \leftarrow \text{pivot} \)
15: \( \text{if} \ \text{start(n,pivot)} > \text{dat(n,pivot)} \lor \text{proc(vip(n))} \neq \text{pivot} \text{ then} \)
16: \( \text{for all} \ \text{p} \in \text{adj(pivot)} \text{ do} \)
17: \( \text{FT} \leftarrow \text{calcstart}(n,p) + w(n,p) \)
18: \( \text{if} \ \text{FT} < \text{finish(n,pivot)} \lor (\text{FT} = \text{finish(n,pivot)} \land \text{proc(vip(n))} = p) \text{ then} \)
19: \( \text{BestFT} \leftarrow \text{FT} \)
20: \( \text{BestProc} \leftarrow \text{p} \)
21: \( \text{end if} \)
22: \( \text{end for} \)
23: \( \text{if} \ \text{p} \neq \text{pivot} \text{ then} \)
24: \( \text{proc(n)} \leftarrow \text{p} \)
25: \( \text{sched}(G,H,\text{proc,n}) \)
26: \( \text{end if} \)
27: \( \text{end if} \)
28: \( \text{end for} \)
29: \( \text{end for} \)
30: \( \text{end function} \)

The rationale for the BSA algorithm is as follows. The authors assume that local data transfers are zero-cost, so the best starting point for the schedule should have the least message transfers. Accordingly, serialinject places all of the vertices on a single processor. All improvements beyond the serial configuration come from moving vertices to adjacent processors where they can start early enough to “hide” the network data transfer cost. As iteration proceeds down the critical path, the migration process reduces the overall potential schedule length with each move of a vertex. The BSA algorithm can be adapted to handle non-uniform data transfers (i.e. different transfer times for different routes) in the calculation of start times for adjacent processing nodes. As the authors
suggest, using the task finish time in the adjacent processor comparisons also adapts the algorithm to
a heterogeneous network with processors running at different speeds[98]. Once the CP and all of its
input dependencies (CP and IB vertices) have been scheduled, the remaining vertices (marked NA)
can be scheduled anywhere convenient in order to reduce network usage or satisfy other objectives.

**Incremental Schedule Analysis**

We will add a few more problem constraints and assumptions appropriate to our incremental variant
of the task graph problem, and adapt the BSA algorithm to meet them:

1. In a real-time system particular tasks are bound to specific processors due to I/O require-
   ments. We will assume that the starting and ending vertex of each critical path correspond
to processor-locked tasks. The problem input will include the assignments for those particular
processors.

2. We assume that an existing schedule has already been placed on the network, and that
the next task graph contains only a single critical path. The pre-existing schedule tasks
are tagged for each critical path in which they participate. For each latency requirement
($L_m$ corresponding to a single critical path $CP_m$), any vertex is either critical ($CP$), an
in-branch dependency (IB), or neither (NA). The resulting vertex list has the form $v_i :$
\{(CP_1, CP), (CP_2, IB), (CP_3, NA), \ldots \}.

3. Any new task graph to be added to the schedule may use existing tasks as part of its speci-
   fication. We refer to the shared vertices as *merge* vertices. However, those composed graphs
may not form a cycle. The full graph is still a DAG with (possibly) multiple input and output
vertices.

4. At the start of the algorithm we have a slack value for each CP in the pre-existing DAG,
which represents an amount by which each CP could expand without violating its end-to-end
deadline.
Concepts

The supported work flow for this algorithm proceeds by scheduling the tasks corresponding to individual requirements sequentially, incrementally adding a DAG with a new CP at each stage as described below. The critical insights are that the initial pre-existing schedule is packed down towards the entry tasks, so any flexibility in the schedule will come from expanding those graphs towards the end of the schedule within the specified slack. That takes a particular form in our approach. If we consider merging two critical paths at specified vertices, then either they are 1) disjoint, 2) meet at a single vertex, 3) meet at sequential vertices (i.e. they share a single common path), or 4) they meet at multiple non-sequential vertices. Case 4 can create problems for our incremental approach. If the CPs meet at multiple non-sequential vertices, then two or more paths exist between each pair. As those multiple paths are shared by both CPs, we may have modified either critical path with a longer segment. This is handled by identifying merge vertices which create multiple paths between CPs. If the path segment from the new CP is longer, then we use slack to expand the schedule distance between those vertices (by moving back the later vertex in the pair). We allow the existing CPs to expand beyond their available slack, but during scheduling all expanded CPs must be packed back down to within their original deadline (positive slack). Once all of the multi-path pairs are expanded towards the end of the schedule, the BSA-style optimization to pack the extra slack back down towards the beginning can proceed.

In BSA, placement of the entry vertex of the new CP is critical, because the BSA algorithm does not ever move the entry vertex in the schedule. Obviously, if the new CP includes existing vertices, then a poor scheduling choice for the initial vertex can cause an infeasible or unnecessarily long schedule. The first segment of the new CP (from the entry vertex to the first merge vertex) will not pack down, because the first merge point is already “packed”. Our solution is to place the entry vertex at the start of the earliest empty segment where the serial injection procedure will reach the merge vertex. Then we use a reverse-BSA algorithm down from the first merge vertex to pack the start of the new CP forward, and use the forward-BSA algorithm to pack the remaining vertices backwards, consuming slack that was given during the initial expansion.
Algorithm Definition

Assume we are given the following inputs, assumptions, and definitions:

- **Existing schedule:** Given a network topology \( H \), an original weighted task graph \( G \), a schedule of start times defined on the vertices of \( G \) \((\text{start}: V(G) \rightarrow \mathbb{Z})\), processor assignments for all of the vertices of \( G \) \((\text{proc}: V(G) \rightarrow [1,P])\), a set of latency requirements \((L_m, m \in [1,M])\), a collection of tags for the vertices (one for each vertex for each requirement) \((\text{tag}: V(G) \times M \rightarrow \{\text{CP}, \text{IB}, \text{NA}\})^{V(G)}\), and a set of slacks for each latency requirement \((\text{slack}: M \rightarrow \mathbb{Z})\).

- **New task graph:** Given a new weighted task graph \( G' \), with latency requirement \( L_{m+1} \) and corresponding critical path \( CP' \). We also have processor assignments for the entry and exit vertices of \( CP' \) \((\text{proc}: v_i, v_f \rightarrow [1,P])\).

- Assume that \( G \cup G' \) is cycle-free.

- Let \( V_s \) and \( E_s \) represent the shared vertices and edges of \( G \cap G' \).

- A vertex \( v \) is a pure free vertex if \( tag(v) = \{(l, NA) | \forall l \in [1,M]\} \). Denote this set \( pfv(G) \).

- Consider \( (G \cup G') - pfv(G \cup G') \). Scheduling excludes pure free vertices in order to obtain maximum scheduling freedom, as all significant (i.e. IB and CP) vertices should be involved in one or more of the critical paths.
Algorithm 4 Incremental BSA Setup

1: function ibsasetup($G, G', H, start, proc, tag, slack$)
2: $CDS \leftarrow \text{SERIALINJECT}(\text{dummy, } G', CP', H)$
3: $pairs \leftarrow \text{FINDFORKS}(G, G', CDS)$
4: for all $(v_1, v_2, cp) \in pairs$ do
5:     Let $v_{1,n} = v | v \in CDS \land (v_1, v) \in E'$.
6:     Let $v_{2,p} = v | v \in CDS \land (v, v_2) \in E'$.
7:     Let $cost = c(v_1, v_{1,n}) + \sum_{v \in CDS, v \in \text{path}(v_1, v_2)} w(v) + c(v_{2,p}, v_2)$.
8:     Let $dist = start(v_2, proc(v_2)) - start(v_1, proc(v_1))$.
9:     $\text{sched}(G, H, proc, v_2)$
10: $slack(cp) \leftarrow slack(cp) - (cost - dist)$
11: end for
12: Let $v = v_1 | (v_1, v_2) \in pairs[1]$
13: return $v$
14: end function

The setup for the incremental BSA algorithm (Algorithm 4) performs the following steps:

- Determination of an initial sequence for $G'$, using a dummy processor with no preexisting task vertices (line 2).

- Calculating the paired vertices of CP' which result in multiple branches when merged with the preexisting CPs (line 3).

- For each pair $(v_1, v_2)$ and associated preexisting CP (indexed by the integer $cp$ in the algorithm):
  - Find the first vertex after $v_1$ in CP' (line 5).
  - Find the last vertex before $v_2$ in CP' (line 6).
  - Calculate the cost of the newly added segment from CP' (line 7).
  - Compare the new segment cost with the distance ($dist$) from $v_1$ to $v_2$ in the current schedule, and the slack for CP. (line 9).
  - If the move is acceptable, move back $v_2$ by $cost - dist$, and reschedule the vertices of CP from that point.

- Return the first vertex of the first pair for splitting the scheduling of CP' into forward and reverse segments (line 14-15).
The incremental BSA (IBSA) algorithm (Algorithm 5) is similar to BSA. The initial processor sorting includes consideration of the fixed processor assignments of the entry and exit vertices of $G'$. The IBSA setup was described previously (Algorithm 4). The reverse and forward progression of the BSA segments proceed from the first merge vertex discovered in the setup routine.

None of the individually modified operations have a worst-case order of operations different from that reported by Kwok and Ahmad[98], except as listed in Table 16. The most significant exception is the modified forward-BSA function ($\text{BSAADV}$), which must process portions of other critical paths when determining whether to move a vertex. In the worst case, the new CP will be merged with all of the other existing CPs, leading to a worst case order of operations as listed. The expected performance gains of the algorithm rely on the small size of the merge sets between the new and existing CPs, so expected values for the order of $\text{BSAADV}$ should be much lower.

**Future Work**

We have presented a conceptual description of a scheduling algorithm that can incrementally calculate task graph schedules. The next step is to implement the scheduler and evaluate its performance for realistic randomized workloads. Unfortunately, there are no standard benchmarks for algorithms in the class of incremental task-graph scheduling algorithms allowing graph merging and
<table>
<thead>
<tr>
<th>Function or Routine</th>
<th>Description</th>
<th>Order of Operations</th>
</tr>
</thead>
<tbody>
<tr>
<td>FINDFORKS</td>
<td>Walk the new CP, finding vertex pairs that cause forks in the merged paths, and their original CP.</td>
<td>(O(em))</td>
</tr>
<tr>
<td>SORTPROCS</td>
<td>Sort the processors according to available time.</td>
<td>(O(pv))</td>
</tr>
<tr>
<td>ISERIALINJECT</td>
<td>Perform SERIALINJECT with a few changes: find the start of the first empty space prior to (merge), and as we move forward, skip over existing vertices. Put all of the (NA) nodes back into the schedule.</td>
<td>(O(e + \nu))</td>
</tr>
<tr>
<td>BSAREV</td>
<td>Perform BSA backward from (merge).</td>
<td>(O(p^2ev))</td>
</tr>
<tr>
<td>BSAADV</td>
<td>Perform BSA forward from (merge), checking CP dependencies for merged CP-nodes before moving them.</td>
<td>(O(p^2evm))</td>
</tr>
</tbody>
</table>

Table 16: Function and variable definitions for incremental BSA.

task-processor binding. Some of the random benchmark graphs described by Kwok and Ahmad could be adapted for this evaluation[33]. The overall schedule length metric is also not wholly appropriate for comparison, unless we consider only the improvement of each latest addition to the schedule.

The scheduler is intended as a replacement for the scheduling analysis algorithm for the ESMoL language originally proposed in Porter et al[70]. The original scheduler created feasible schedules, but had difficulty enforcing end-to-end latency constraints consistently. This scheduler includes allocation and scheduling, which makes the integration prospects more interesting for ESMoL. The next step is integration with the ESMoL scheduling specification language.

If necessary to improve scalability, we should consider encoding some of the useful task graph scheduling abstractions as constraint problems. Earlier work in constraint programming for schedule calculation shows that constraint-based techniques scale very well to large problems with many dependencies [34][70]. BSA is essentially a search over multiple dimensions, with interval narrowing of the start times towards the beginning of the schedule. It should be possible to investigate this approach to see if it can yield a more scalable version of the algorithm.
CHAPTER VI

CONCLUSIONS: WHAT HAVE WE LEARNED?

The ESMoL language and tools provide sufficient expressiveness and detail to analyze and generate functional quadrotor control software for deployment on a time-triggered distributed processing network. The reach of our model-integrated tool environment was not sufficient to capture all of the difficult details involved in our modeling example, so further work on library integration for tool development and runtime evaluation are particularly important.

Model structure can facilitate the specification and implementation of incremental syntactic model analysis. Our example offers a proof of concept, and the expectation that where system behavior can be represented compositionally according to the hierarchical structure of the model, such techniques will prove beneficial. Further work will determine scalability of our analysis approach, though conceptually the incremental approach already increases the flexibility of the development process.

For purely semantic properties, a proper choice of behavioral abstractions can permit incremental analysis as well. Our scheduling approach is only conceptual, but illustrates the richness of this line of inquiry. Further work will implement and evaluate this potentially useful technique, in order to assess scalability of the algorithm and conservatism of the results produced by it.
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