MONETARY POLICY, TRADE PATTERNS, AND PRICE DISPERSION:

THE ROLE OF TRADE COSTS

By

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# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>ACKNOWLEDGMENTS</th>
<th>iii</th>
</tr>
</thead>
<tbody>
<tr>
<td>LIST OF TABLES</td>
<td>vii</td>
</tr>
<tr>
<td>LIST OF FIGURES</td>
<td>ix</td>
</tr>
</tbody>
</table>

**Chapter**

I INTRODUCTION ........................................................................................................ 1
   - Is there a Role for Trade Costs in Explaining the Central Bank Behavior?..... 1
   - Understanding Interstate Trade Patterns ............................................. 3
   - A Model of International Cities: Implications for Real Exchange Rates ....... 5

II IS THERE A ROLE FOR TRADE COSTS IN EXPLAINING THE CENTRAL BANK BEHAVIOR? 9
   - Introduction............................................................................................... 9
   - The Model................................................................................................. 13
   - Estimation.................................................................................................. 18
     - Estimation of the Monetary Policy Rule............................................ 18
     - Estimation of the New-Keynesian Phillips Curve............................... 21
     - Remaining Parameters..................................................................... 22
   - Results and Comparisons............................................................................ 22
     - Utility-Based Loss Function ............................................................. 23
     - Ad Hoc Loss Function...................................................................... 25
       - Exercise 1.......................................................................... 27
       - Exercise 2.......................................................................... 27
       - Exercise 3.......................................................................... 28
     - Impulse Response Functions............................................................. 31
   - Conclusions................................................................................................ 31
   - Appendices................................................................................................ 45
     - A. Definitions and Some Identities.................................................... 45
     - B. Individuals.................................................................................. 48
     - C. Firms......................................................................................... 51
     - D. Aggregate Demand...................................................................... 52
     - E. The New-Keynesian Phillips Curve............................................... 53
     - F. Equilibrium Dynamics .................................................................. 55
     - G. Utility-Based Welfare .................................................................. 57
     - H. Model Solution ............................................................................ 59
     - I. Data Appendix ............................................................................. 62

III UNDERSTANDING INTERSTATE TRADE PATTERNS ........................................ 64
   - Introduction............................................................................................... 64
   - The Model................................................................................................. 69
<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Implications for Wages, Rents, Wage Income, and Capital</td>
<td>167</td>
</tr>
<tr>
<td>Implications for Price Ratios across Cities</td>
<td>171</td>
</tr>
<tr>
<td>Estimation Appendix</td>
<td>172</td>
</tr>
<tr>
<td>BIBLIOGRAPHY</td>
<td>173</td>
</tr>
</tbody>
</table>
## LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>33</td>
</tr>
<tr>
<td>2</td>
<td>34</td>
</tr>
<tr>
<td>3</td>
<td>35</td>
</tr>
<tr>
<td>4</td>
<td>36</td>
</tr>
<tr>
<td>5</td>
<td>37</td>
</tr>
<tr>
<td>6</td>
<td>38</td>
</tr>
<tr>
<td>7</td>
<td>98</td>
</tr>
<tr>
<td>8</td>
<td>99</td>
</tr>
<tr>
<td>9</td>
<td>103</td>
</tr>
<tr>
<td>10</td>
<td>107</td>
</tr>
<tr>
<td>11</td>
<td>108</td>
</tr>
<tr>
<td>12</td>
<td>109</td>
</tr>
<tr>
<td>13</td>
<td>110</td>
</tr>
<tr>
<td>14</td>
<td>111</td>
</tr>
<tr>
<td>15</td>
<td>112</td>
</tr>
<tr>
<td>16</td>
<td>113</td>
</tr>
<tr>
<td>17</td>
<td>114</td>
</tr>
<tr>
<td>18</td>
<td>115</td>
</tr>
<tr>
<td>19</td>
<td>118</td>
</tr>
<tr>
<td>20</td>
<td>151</td>
</tr>
<tr>
<td>Section</td>
<td>Page</td>
</tr>
<tr>
<td>------------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>Mean Sectoral Wage Differentials</td>
<td>152</td>
</tr>
<tr>
<td>Variance of Wage Differentials across Sectors and Locations</td>
<td>153</td>
</tr>
<tr>
<td>Explanatory Power</td>
<td>154</td>
</tr>
<tr>
<td>Variance of Prices across Locations</td>
<td>155</td>
</tr>
<tr>
<td>Variance Decomposition (median across EIU goods)</td>
<td>156</td>
</tr>
<tr>
<td>Variance Decomposition (aggregate NIPA)</td>
<td>157</td>
</tr>
</tbody>
</table>
# LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Expected Loss Values for Historical MPR in the presence of Trade Costs</td>
<td>39</td>
</tr>
<tr>
<td>2</td>
<td>Expected Loss Values for Historical MPR in the absence of Trade Costs</td>
<td>39</td>
</tr>
<tr>
<td>3</td>
<td>Expected Loss Values for Optimal MPR in the presence of Trade Costs</td>
<td>40</td>
</tr>
<tr>
<td>4</td>
<td>Optimal Coefficient of Inflation in the presence of Trade Costs</td>
<td>40</td>
</tr>
<tr>
<td>5</td>
<td>Optimal Coefficient of Output in the presence of Trade Costs</td>
<td>40</td>
</tr>
<tr>
<td>6</td>
<td>Optimal Coefficient of Interest Rate in the presence of Trade Costs</td>
<td>41</td>
</tr>
<tr>
<td>7</td>
<td>Expected Loss Values for Optimal MPR in the absence of Trade Costs</td>
<td>41</td>
</tr>
<tr>
<td>8</td>
<td>Optimal Coefficient of Inflation in the absence of Trade Costs</td>
<td>41</td>
</tr>
<tr>
<td>9</td>
<td>Optimal Coefficient of Output in the absence of Trade Costs</td>
<td>42</td>
</tr>
<tr>
<td>10</td>
<td>Optimal Coefficient of Interest Rate in the presence of Trade Costs</td>
<td>42</td>
</tr>
<tr>
<td>11</td>
<td>Percentage Deviation from Optimal Expected Loss in the Presence of Trade Costs</td>
<td>42</td>
</tr>
<tr>
<td>12</td>
<td>Percentage Deviation from Optimal Expected Loss in the Absence of Trade Costs</td>
<td>43</td>
</tr>
<tr>
<td>13</td>
<td>Percentage Deviation from Optimal Expected Loss for the Hybrid Case</td>
<td>43</td>
</tr>
<tr>
<td>14</td>
<td>Response of Output Gap</td>
<td>43</td>
</tr>
<tr>
<td>15</td>
<td>Response of Inflation</td>
<td>44</td>
</tr>
<tr>
<td>16</td>
<td>Response of Nominal Interest</td>
<td>44</td>
</tr>
<tr>
<td>17</td>
<td>Response of Real Exchange Rate</td>
<td>44</td>
</tr>
<tr>
<td>18</td>
<td>Kernel Density Estimates of Price Distributions</td>
<td>158</td>
</tr>
<tr>
<td>19</td>
<td>Variance Decomposition as a Function of Traded Input Share</td>
<td>159</td>
</tr>
<tr>
<td>20</td>
<td>Variance Decomposition as a Function of Non-traded Labor Input Share</td>
<td>160</td>
</tr>
</tbody>
</table>
CHAPTER I

INTRODUCTION

This dissertation consists of three studies. Each study examines the role of trade costs for some facet of trade or finance, thus partly addressing a number of the Six Puzzles of International Economics highlighted by Obstfeld and Rogoff (2000). The first one studies inflation targeting by the Bank of Canada, in the context of a small open economy subject to trade costs. The second one develops a partial equilibrium gravity model and estimates key trade and trade cost elasticities, previously not separately identified. The final one develops a general equilibrium trade model across cities with traded goods and a distribution sector in each city. We explain the details of each study in the following sections.

Is there a Role for Trade Costs in Explaining the Central Bank Behavior?

The goal of this study is to address the following question: What is the role of trade costs in monetary policy? This question is important, because as is well known, the actions of a central bank play a big role in the overall performance of an economy. More specifically, a sound economic environment (in which the inflation rate should be low) can be obtained only through accurate economic policies determined by central banks. Otherwise, an economy can experience devastating business cycles just like the U.S. economy had experienced during the Great Depreciation or World War II. In order to determine which monetary policy is right and which monetary policy is wrong, a central bank authority has to consider what is going on in the economy. The situation of an economy can be
determined through its dynamics mainly consisting of two agents: individuals and firms. Since the behaviour of these agents is affected by trade costs, the central bank needs to take trade costs into account when framing its policy decisions.

The role of trade costs has not been considered in the literature when evaluating the performance of central banks. This is surprising given the long-standing emphasis of trade costs in international trade and finance models. This study is intended to bridge the gap and show that trade costs may play an important rule in the actions of central banks. We investigate whether or not central banks, especially the Bank of Canada, take trade costs into account when they alter their policy instruments. Canada is chosen because it is a small open economy, open to shocks in trade costs, especially the shocks in oil prices. Approximately 36% of consumption in Canada is imported. Moreover, Canada has explicitly targeted inflation since 1991. In sum, Canada is an appropriate case study given the focus of the model.

Turning to the details, the model boils down to two main equations, an IS curve (an indicator for the behaviour of individuals) and a New-Keynesian Phillips curve (an indicator for the behaviour of firms). These two equations help define the current economic situation and the policy reaction function of the central bank.

We estimate these equations using the Generalized Method of Moments (GMM) and use the resulting equations to consider both the welfare effects of existing policy action by the Bank of Canada and to evaluate historical policy in comparison to optimal policy, conditional on the model. When a utility-based expected loss function is considered, the central bank is found to be far from being optimal in its actions, independent of trade costs. This suggests that a utility-based expected loss function may not be what the central bank of Canada uses when deciding on monetary policy. When an ad hoc expected loss function
considering the volatilities in inflation, output and interest rate is considered, it is found that the actions of the central bank are explained best when trade costs in fact exist but the central bank ignores them. Given the ad hoc loss function, the actions of the central bank are best explained when 70% of weight is assigned to inflation, 15% of weight to interest rate and 15% of weight to output.

Understanding Interstate Trade Patterns

What is the main motivation behind intranational trade? Compared to relatively complex models in the literature, this study contributes to the debate using a partial equilibrium model to analyze the motivations behind bilateral trade patterns of regions within the U.S. at the disaggregated level. We attempt to find why regions import more goods from some regions while importing less from others. The main determinants of trade in the model are geography, distance and technology differences.

In particular, we study a monopolistic competition model consisting of a finite number of regions with two types of goods, traded and non-traded. Each region produces and consumes a unique non-traded good. Each region may also consume all varieties of all traded goods while producing one variety of each traded good. While the traded goods are produced by a perfectly mobile unique factor, the only non-traded good in each region is produced by the same mobile factor together with traded intermediate inputs.

We show that the trade of a variety of a particular traded good across any two regions depends on the relative price of the variety and the total demand (final consumption demand plus intermediate input demand) of the good in the destination (importer) region. This is standard. The contribution comes into the picture when the ratio between imports of varieties from different origins (exporters) are considered.
We show that the estimated model offers numerous insights:

- There is no identification problem in terms of separately estimating the elasticity of substitution across varieties of a good and the elasticity of distance at the same time.

- The methodology avoids possible upward bias in the distance measures (due to using calculated distances, such as great circle) mentioned by Hillberry and Hummels (2001).

- The model is also capable of controlling for the effects of local (i.e., wholesale and retail) distribution costs, insurance costs, local taxes, international trade (under reasonable assumptions), and intermediate input trade, each of which are heavily debated and largely unresolved in the existing literature (see Anderson and van Wincoop 2004).

- The analytical solution for bilateral trade flows obviates the need for income data given the technology levels.

The model is estimated using bilateral trade data across U.S. states. The estimated parameters correspond to: a) the elasticity of substitution across varieties of a good, each produced in a different region; b) the elasticity of distance, which governs good specific trade costs; and c) the heterogeneity of individual tastes, such as ‘home-bias.’ Several empirical strategies are pursued and sensitivity analysis is conducted. Overall, the model is capable of explaining the interstate trade data up to 60% at the disaggregate level.

The estimated parameters provide insights about U.S. interstate trading patterns that mostly contrast with international patterns: a) compared to empirical international studies, the elasticity of substitution is lower intranationally; b) compared to empirical international studies, the elasticity of distance is higher intranationally; c) there is evidence for home-bias even at the intranational level; d) trade costs are mostly good specific even at the intranational level; e) source specific fixed effects are important for bilateral trade
patterns, which is usually ignored in the literature; f) production technologies are both good and region specific rather than country specific.

**A Model of International Cities: Implications for Real Exchange Rates**

This study develops and estimates a novel theory of trade which extends the prototype trade model to include a retail and distribution market sector. The competitive trade model assumes that goods are shipped from the foreign factory gate to the final consumer; as such the price difference across two locations equals a shipping cost plus a distortion due to trade policy, such as an import tariff, unless the good is duty free. This extension to existing theory is necessary for two reasons.

The first reason to extend existing trade models is driven by empirical evidence. The observed deviations of prices across locations at the level of retail goods and services have been shown to exceed what may be reasonably attributed to shipping costs and tariff barriers. That is, the magnitudes of international shipping costs and tariff barriers are insufficient to account for the bulk of international price dispersion that we observe. This is particularly evident when the less developed nations are brought into the analysis.

The second reason is more theoretical. Analysis to date has focused on highly traded manufactured goods, which represent less than 20 percent of employment and gross national product while ignoring the service and distribution sector. Thus, the existing models and empirical studies have omitted the bulk of economic activity as well as the economic interactions that take place across traded and non-traded goods. For example, medical services are largely non-traded and yet used traded goods as inputs. Thus the interaction of the traded and non-traded sectors is an important dimension of price determination and trade flows.
The model follows the existing literature in assuming specialization in the manufacturing sector. Since we are referring to a single good and city given the empirical focus of the work, this specialization assumption is more innocuous in this setting than when it is imposed in national level analysis at higher levels of aggregation across sectors. That is, it is more plausible that a region of a country supplies much of the world market with a brand of a particular good than it is for a nation to dominate the world market in an entire sector. By choosing to disaggregate more in the good and location dimension, our model will have some of the flavour of the increasingly popular models of variety.

The more novel element is to have a second economic agent in each city (the first is the producer of the traded good) that effectively shops the world for the best deals and offers all the goods for sale in the local retail market. This brings in the distribution sector as a time allocation problem at the local level and allows us also to capture both retail services and labour allocated to locally produced services such as medicine and education. Thus, if much of the local available pool of labour is allocated to services, less labour is available to produce traded goods for the purpose of international exchange. The model structure captures the direct and indirect economic interactions between traded and local inputs at the level of individual goods and services in a general equilibrium context.

We incorporate productivity at the city level in both the retail sector and the individual good that the city exports to other cities. Thus each location has two productivity variables, one that affects its efficiency in production of the traded good and one that affects its efficiency across the entire retail sector. The former will alter the price of a good relative to all other goods in the consumption basket in a similar way across locations; the latter will alter the relative price level (e.g. the CPI index) across locations. These two margins allow the model to capture more of the price dispersion at the retail level than models with
a single sector or only two production locations.

The data used to quantify the implications of theory as well as test its validity are the Economist Intelligence Unit worldwide surveys of retail prices. The data contains price information about 301 goods and services in 123 international cities annually between 1990 and 2005. This is the best available data that includes price data from 79 different countries as well as data from cities within the same countries. In this way, we can compare the price dispersion across countries (e.g., the US vs. Canada) and within countries (e.g., New York vs. Los Angeles) empirically after controlling for exchange rate differences. The intranational dimension allows us to isolate international commercial policy and exchange rate effects from factors that drive price differences both within and across countries, such as the distribution sector, shipping costs and markups. Markups are the price margins retailers charge over and above the marginal cost of the good. In our competitive model they are ignored, in our empirical work we treat them as residuals. Thus, we are not attempting to explain all of price dispersion with the competitive model we develop, but the fraction not attributable to markup differences across locations.

What we explain is a large fraction of what we see in the data, however. The empirical results show that our model is capable of explaining the price dispersion across 123 cities up to 80%, the remaining is attributed to markups and measurement error. This suggests that price differences are indeed due to trade costs and retail costs, a very promising result for the modelling approach we take.

Parsing this explained variation into trade costs and location specific retail costs, we find their relative contribution depends on the geography of locations pooled together. That is, if we take locations within the same country as opposed to cross-border pairs, the variance and its component decomposition changes dramatically. For cross-border pairs
(respectively, locations within the same country), on average, we find trade costs account for 51 (respectively, 56) percent of the variance, distribution costs account for about 11 (respectively, 10) percent. Since the micro-data we use is skewed toward traded goods, we also decompose the variance for cross-border pairs (respectively, locations within the same country) based on the median good on an expenditure weight based; the tables turn, with distribution accounting for 43 (respectively, 15) percent and trade costs 36 (respectively, 60) percent.
CHAPTER II

IS THERE A ROLE FOR TRADE COSTS IN EXPLAINING THE CENTRAL BANK BEHAVIOR?

Introduction

Research on inflation targeting and monetary policy has focused on explaining the actual central bank behavior.\(^1\) But, is there a role for trade costs in explaining this behavior? This chapter attempts to answer this question using a dynamic stochastic general equilibrium model (DGSE) with the addition of trade costs shocks to an otherwise standard New-Keynesian model of the Bank of Canada monetary policy. A New-Keynesian Phillips curve, together with the monetary policy rule of the Bank of Canada, is estimated for the Canadian economy. In order to analyze the effects of trade costs on monetary policy, we consider versions of the model, with and without trade costs. It is found that, under a utility-based expected loss function, the Bank of Canada appears to be far from optimal in its actions, independent of trade costs. In contrast, under an \textit{ad hoc} expected loss function, the actions of the Bank of Canada are explained best when trade costs in fact exist, but the Bank of Canada ignores them. We also show that, given the ad hoc loss function, the actions of the Bank of Canada are best explained when 70\% of weight is assigned to inflation, 15\% of weight to interest rate and 15\% of weight to output.

We set up an open economy model with the home country and the rest of the world. In the model, there are three sets of agents: individuals, firms, and the central bank policy makers. Individuals maximize their intertemporal lifetime expected utility function consisting of utility obtained from domestic (home) goods and imported goods, together with disutility from supplying labor. The production of goods requires labor input combined with technology. The model employs a Calvo price-setting process, in which firms are able to change their prices only with some probability, independent of other firms and the time elapsed since the last adjustment. Firms behave as monopolistic competitors. Imported final goods are subject to a trade cost for both domestic individuals and foreign individuals. The main nuance of the model is the inclusion of trade costs which is important in terms of its implications on real exchange rates and the Law-of-One-Price.\(^2\)

The micro-foundations of the individual-firm behavior results in the IS curve and the New-Keynesian Phillips curve. While the New-Keynesian Phillips curve takes into account the non-zero inflation target as the steady-state inflation (similar to the studies such as Kozicki and Tinsley, 2003; Ascari, 2004; Cogley and Sbordone, 2006; Amano et al., 2006, 2007; Bakhshi et al., 2007; Sbordone, 2007), the IS curve considers the effect of trade costs on output, which is not the usual case in the literature.\(^3\) In particular, we find that the output decreases with trade costs. Moreover, an expected increase in trade costs has a negative effect on the expected change in output gap, \textit{ceteris paribus}. For the monetary policy rule, we assume that the central bank manages a short-term nominal interest rate according to an open economy variant of the Taylor rule. Following Yazgan


and Yilmazkuday (2007), we modify the monetary policy rule of Taylor (1993) and Clarida et al. (1998, 1999, and 2000) by keeping the inflation target in the final form of the rule.

Another contribution of this chapter is the estimation of the New-Keynesian Phillips curve, together with the monetary policy rule for the Canadian economy, by using the Generalized Method of Moments (GMM). However, recently, GMM estimators have been criticized on the grounds that inference, based on these estimators, is inconclusive. The related econometric literature indicates that there has been considerable evidence that asymptotic normality provides a poor approximation of the sampling distributions of GMM estimators. Particularly, the GMM estimator becomes heavily biased (in the same direction as the ordinary least squares estimator), and the distribution of the GMM estimator is quite far from the normal distribution (e.g. bimodal). Stock and Wright (2000) attribute this problem to “weak identification” or “weak instruments,” that is, instruments that are only weakly correlated with the included endogenous variables. Stock et al. (2002) and Dufour (2003) provide a comprehensive survey on weak identification in GMM estimation. In this chapter, we address the problem of weak identification by using two different tests. The first of these tests is the Anderson and Roubin (1949) test ($AR$-test) in its general form presented by Kleibergen (2002). The second test is the $K$-test developed by Kleibergen (2002). These two tests are robust in the case of nonlinear models (see Dufour, 2003; Stock et al., 2002), and perhaps more importantly, they are robust even to excluded instruments (see Dufour, 2003). Since it is rarely possible to use all possible instruments, this latter property is quite important from an applied point of view (see Yazgan and Yilmazkuday, 2005, 2007).

By applying a simulation based on the estimated parameters, we find optimal monetary policy rules under different scenarios. In particular, following the lead of Ambler et al. (2004), Cayen et al. (2006), Murchison and Rennison (2006), Ortega and Rebei
(2006), which give insights about the Bank of Canada’s policy-analysis models, we use the method of stochastic simulations to determine the vector of monetary policy rule parameters that minimizes the expected loss function, given the dynamics of the Canadian economy (i.e., the IS curve and the estimated New-Keynesian Phillips curve).\footnote{Also see Tetlow and von zur Muehlen (1999), Erceg et al. (1998, 2000) as other studies on optimized monetary policy rules.} Following Woodford (2003), we first consider a utility-based expected loss function and show that the Bank of Canada is far from being optimal in such a case, independent of trade costs.

We then consider an \textit{ad hoc} expected loss function and compare the calculated optimal monetary rules with the estimated monetary policy rule to obtain the weights assigned to inflation, output and interest rate volatilities, at which the percentage deviation of the expected loss from its optimal value takes its minimum value. We follow an optimistic approach and accept these calculated weights as the Bank of Canada’s policy weights. Thus, instead of assigning specific weights to the mentioned variables in the loss function, we calculate them by simulation techniques.\footnote{See Rotemberg and Woodford (1997); Woodford (1999); Batini and Nelson (2001); Smets (2003); Parrado (2004); Yilmazkuday (2007), among many others, for different types of loss functions considered in the literature.} The simulation results show that the actions of the Bank of Canada are best explained when trade costs actually exist but the Bank of Canada ignores them.

The rest of this chapter is organized as follows: Section II introduces the New-Keynesian model and illustrates our modified specification of monetary policy developed to take into account the inflation targets. Section III presents the main estimation results. Section IV depicts the results and comparisons of the simulation based on the Canadian economy. Section V concludes. The derivation of the model, together with the details of
the data used, is given in the Appendices.

The Model

Extending a simpler version of Gali and Monacelli’s (2005) model by introducing trade costs, we introduce a continuum of goods model in which all goods are tradable, the representative individual holds assets, and the production of goods requires only labor input. The optimality conditions of the agents (i.e., microfoundations of the model) are derived in the Appendices, and the key equations are introduced in the text. Although the text of this chapter is self-contained, the reader is encouraged to read the Appendices and the footnotes there for very important technical details of this chapter.

In the model, the aggregate demand is as follows:

\[ y_t = E_t (y_{t+1}) - (i_t - E_t (\pi_{H,t+1})) + E_t (\Delta \tau_{t+1}) \]  

(II.1)

where \( y_t \) is the (log) output; \( i_t \) is nominal interest rate; \( \tau_t \) is the (log) gross trade cost; and \( \pi_{H,t} \) is the inflation of home-produced goods. In particular, Equation (II.1) represents an IS curve that considers the effect of trade costs on output, which is not the usual case in the literature where the third term (i.e., the change in trade costs) is absent. The derivation of Equation (II.1) is given in Appendix D.

From another point of view, Equation (II.1) represents an IS curve that relates the expected change in (log) output (i.e., \( E_t (y_{t+1}) - y_t \)) to the difference between the interest rate, the expected future domestic inflation (i.e., an approximate measure of real interest rate that becomes an exact measure of real interest rate when the terms of trade are constant across periods), and the expected change in trade costs.\(^6\) An increase in

\(^6\)See Kerr and King (1996), and King (2000) for discussions on incorporating the role for future output gap in the IS curve with a unit coefficient.
the difference between the expected inflation and the nominal interest rate decreases the expected change in the output gap, with a unit coefficient. Finally, an expected increase in the trade costs leads to a decrease in the expected change in (log) output. The latter is due to the intertemporal substitution of supply in response to a change in trade cost.

As is also shown in Appendix F, Equation (II.1) can be written in terms of output gap as follows:

\[
x_t = E_t (x_{t+1}) - (i_t - E_t (\pi_{H,t+1})) + E_t (\Delta z_{t+1})
\]  

(II.2)

where \( x_t = y_t - \bar{y}_t \) is the output gap defined as the deviation of (log) domestic output \( y_t \) from the domestic natural level of output \( \bar{y}_t \) defined as the one under flexible price equilibrium; and \( z_t \) is the (log) level of technology, which evolves according to:

\[
z_t = \rho_z z_{t-1} + \varepsilon^z_t \]  

(II.3)

where \( \rho_z \in [0, 1] \) and \( \varepsilon^z_t \) is assumed to be an i.i.d. shock with zero mean and variance \( \sigma^2_z \).

The New-Keynesian Phillips curve in this economy is given by:

\[
\pi_{H,t} = \lambda_{\pi} E_t [\pi_{H,t+1}] + \lambda_m (\varphi + \hat{m}c_t)
\]  

(II.4)

where \( \lambda_{\pi} = \frac{\beta \alpha}{1 - (1 - \alpha)(\Pi)} \), \( \lambda_m = \frac{(1 - \alpha)(1 - \alpha \beta)}{1 - (1 - \alpha)(\Pi)} \), and \( \varphi = 1 - \Pi (1 - \pi) \). In particular, \( \alpha \) is the probability that a firm does not change its price within a given period; \( \beta \) is the discount factor; \( \Pi = \exp (\pi) \) is the exponential of trend inflation; and \( \hat{m}c_t = mc_t - mc \) is the log deviation of real marginal cost from its steady state value. Note that this expression reduces to a standard New-Keynesian Phillips curve when trend inflation is equal to zero (i.e., \( \pi = 0 \) or \( \Pi = 1 \)). The derivation of Equation (II.4) is given in Appendix E.

For the monetary policy rule, we assume that the central bank manages a short-term nominal interest rate according to the Taylor rule. Following Taylor (1993) and Clarida
et al. (1998, 1999, and 2000), the monetary policy rule is given by:

\[ \tilde{i}_t = r + \tilde{\pi} + \chi_\pi \left[ E_t (\pi_{t+1} | \Omega_t) - \tilde{\pi} \right] + \chi_x E_t (x_t | \Omega_t) \]  

(II.5)

where \( \tilde{i}_t \) denotes the target rate for nominal interest rate in period \( t \); is the information set at the time the interest rate is set; \( \pi_{t+1} \) denotes CPI inflation one period ahead; \( \tilde{\pi} \) is the target for CPI inflation; \( x_t \) is the output gap in period \( t \); and \( r \) is the long-run equilibrium real rate.\(^7\) As in Clarida et al. (2000), we assume that the real rate is stationary and is determined by non-monetary factors in the long run. Since we consider the monthly sample over the period 1996:1 to 2006:12, in which the annual inflation target range is exactly the same (i.e., 2%, the midpoint of a control range of 1% to 3%, according to the Bank of Canada, Macklem, 2002, and Coletti and Murchison, 2002) and the long-run interest rates are pretty much stable for the Canadian economy, we assume \( r \) and \( \tilde{\pi} \) are time invariant.

Similar policy rules to (II.5) have been used in empirical research of several countries. However, most of these and previously mentioned studies consider a zero inflation target over the period of estimation. In this study, following the lead of Yazgan and Yilmazkuday (2007), we keep the inflation target in the monetary policy rule and modify Equation (II.5) as follows:

\[ i_t = r + \tilde{\pi} + \chi_\pi \left[ \pi_{t+1} - \tilde{\pi} \right] + \chi_x x_t + \psi_t \]  

(II.6)

where \( i_t \) is the actual nominal interest rate, and

\[ \psi_t = -\chi_\pi \left[ \pi_{t+1} - E_t (\pi_{t+1} | \Omega_t) \right] - \chi_x [x_t - E_t (x_t | \Omega_t)] + \mu_t \]

\(^7\)It should be noted that \( r \) is an “approximate” real rate since the forecast horizon for the inflation rate will generally differ from the maturity of the short-term nominal rate used as a monetary policy instrument. As noted by Clarida et al. (2000), in practice, the presence of high correlation between the short-term rates at maturities associated with the target horizon (1 year) prevents this from being a problem.
The term \( \mu_t \) captures the difference between the desired and the actual nominal interest rate, i.e. \( \mu_t = i_t - \bar{i}_t \).

According to Clarida et al. (2000), this difference may result from three sources. First, the specification in Equation (II.6) assumes an adjustment of the actual overnight rates to its target level, and thus ignores, if any, the Bank of Canada’s tendency to smooth changes in interest rates (we will address this issue below). Second, it treats all changes in interest rates over time as reflecting the Bank of Canada’s systematic response to economic conditions. Specifically, it does not allow for any randomness in policy actions, other than that which is associated with misforecasts of the economy. Third, it assumes that the Bank of Canada has perfect control over the interest rates, i.e., it succeeds in keeping them at the desired level (e.g., through open market operations).

Interest rate smoothing is introduced into the model via the following partial adjustment mechanism (see Clarida et al., 1998, 2000):

\[
i_t = (1 - \rho_i) \bar{i}_t + \rho_i i_{t-1} + \nu_t
\]  

(II.7)

where \( \rho_i \in [0, 1] \) captures the degree of interest rate smoothing. Equation (II.7) postulates that in each period, the Bank of Canada adjusts the funds rate to eliminate a fraction \((1 - \rho_i)\) of the gap between its current target level and its past value. And, \( \nu_t \) is an independently and identically distributed error term. Substituting Equation (II.5) into Equation (II.7) yields:

\[
i_t = (1 - \rho_i) \left( r + \bar{\pi} + \chi_\pi [\pi_{t+1} - \bar{\pi}] + \chi_x x_t \right) + \rho_i i_{t-1} + \varepsilon_t
\]  

(II.8)

where \( \varepsilon_t = - (1 - \rho_i) \left\{ \chi_\pi [\pi_{t+1} - E_t (\pi_{t+1} | \Omega_t)] + \chi_x [x_t - E_t (x_t | \Omega_t)] \right\} + \nu_t.\)

\(^8\)We assume that \( \mu_t \) is identically and independently distributed.
Using the utility function specified in the model, we may define the utility-based welfare function as follows:

\[
E_t \sum_{k=0}^{\infty} \beta^{t+k} (U(C_{t+k}) - V(N_{t+k}))
\]

\[
= E_t \sum_{k=0}^{\infty} \beta^{t+k} \left( \log \zeta + \tau_{t+k} - \gamma \right) s_{t+k}
\]

\[-\frac{1}{2} E_t \sum_{k=0}^{\infty} \beta^{t+k} \left( (1 - \log \zeta - \tau_{t+k}) \frac{\theta}{\lambda_w} (\pi_{H,t+k})^2 + (x_{t+k})^2 \right)
\]

\[+ t.i.p + o(\|a^3\|)\]

where \(\lambda_w = \frac{(1-\alpha)(1-\alpha \bar{a})}{\alpha}\); \(\zeta \equiv \theta/(\theta - 1)\) is a markup as a result of market power; \(\theta > 1\) is the price elasticity of demand faced by each monopolist; \(s_t\) is the (log) effective terms of trade defined as the difference between foreign and domestic prices; \(\gamma\) is the share of domestic consumption allocated to imported goods; \(t.i.p.\) represents terms independent of policy; and finally, \(o(\|a^3\|)\) represents terms that are equal to or higher than 3rd order. The derivation of the utility-based welfare function is shown in Appendix F.

Note that the utility-based welfare function depends on the volatility in inflation and output gap as well as the trade costs and the terms of trade. It is derived explicitly as a quadratic approximation to the utility function of the representative household. However, the welfare comparisons below are made on the basis of a linearized model. We know on the results of Kim and Kim (2003) that this can be misleading, because linear approximate methods fail to take into account the impact of uncertainty (stochastic shocks) on the expected values of the endogeneous variables. In order to remedy this problem, following Erceg et al. (2000), we introduce taxes and subsidies into our model such that the steady state of the economy is Pareto optimum. See the Appendices for the details.
In our initial welfare analysis, we assume that the Bank of Canada is benevolent and thus uses Equation (II.9) as its objective function. We will relax this assumption later to consider an alternative \textit{ad hoc} objective function. For now, we need parameter values of $\alpha$, $\beta$, and $\theta$ to calculate the value of the welfare function of Equation (II.9). We estimate these parameters in the next section.

**Estimation**

In this section, we separately estimate the monetary policy rule of the Bank of Canada and the New-Keynesian Phillips curve for the Canadian economy by using continuous updating GMM.\footnote{Our reason for individual GMM estimations is that joint GMM estimations can be hazardous according to Hayashi (2000, p.273). In particular, while a joint estimation theoretically provides asymptotic efficiency, it may suffer more from the small-sample bias in practice.} Our estimation results will not only help us determine how our model explains the Canadian data, but they will also provide parameters for our simulation analysis in the next section. The data are described in Appendix I.

**Estimation of the Monetary Policy Rule**

Let $zz_t$ be a vector of variables, within the central bank’s information set at the time it chooses the interest rate (i.e. $zz_t \in \Omega_t$) that are orthogonal to $\varepsilon_t$. Possible elements of $zz_t$ include any lagged variables that help to forecast inflation and output gap, as well as any contemporaneous variables that are uncorrelated with the current interest rate shock $\mu_t$. In sum, we have the following orthogonality condition:

$$E_t [i_t - (1 - \rho) \{r + \hat{\pi} + \chi_{\pi} [\pi_{t+1} - \bar{\pi}] + \chi_x x_t\} - \rho \mu_{t-1} | zz_t] = 0 \quad (\text{II.10})$$

In Equation (II.10), the expected signs of $r, \rho, \chi_{\pi}, \chi_x$ are all positive. By using
this orthogonality condition, we use continuous updating GMM to estimate the parameter vector \([r, \rho, \chi_\pi, \chi_x]\).\(^{10}\) Since the econometric estimation procedure that we use here (GMM) requires that all the variables (including instruments) used in the estimation should be stationary, all of the variables are tested by using the Augmented Dickey-Fuller (ADF) tests. We find that the null of unit root is rejected in all variables, at least at the 10 percent significance level, when tests are applied at different lags.\(^{11}\) The results are illustrated in Table 1. The instruments we use for GMM estimation consist of twelve lags of home inflation, percentage change in M1 and three lags of output gap.\(^{12}\)

Table 1 reports the estimates of \(r, \chi_\pi, \chi_x\) and \(\rho_i\). All of the estimates satisfy their expected signs.\(^{13}\) In particular, the estimate of the coefficient on the difference between expected and targeted inflation is around 5.50 for Canada. That is, if expected inflation were 1 percentage point above the target, the Bank of Canada would set the interest rate approximately 5.50 percent above its equilibrium value. This coefficient is significant at the 10% level when we use asymptotic normality as an approximation to the sampling distribution of GMM estimators.

The response of the Bank of Canada to the deviations of the expected output gap from its target (assumed to be zero) is around 0.09. In other words, holding other parameters constant, one unit increase in output gap induces the Bank of Canada to increase the interest rates by 9 basis points. This coefficient is again significant at the 10% level. The equilibrium real interest rate is estimated as 1.37 percent and it is significant at the 10% level using

\(^{10}\)For continuous updating GMM estimators, we have modified the GAUSS code originally used by Stock and Wright (2000). All of our codes are available upon request. Gauss version 6.0 has been used.

\(^{11}\)These results are available upon request.

\(^{12}\)By choosing these instruments, we implicitly assume that these variables are strong instruments for predicting inflation and output gap.

\(^{13}\)Although the comparison of these estimates with the existing literature is difficult due to the differences in model specifications and sample periods, see Ambler et al. (2004), Murchison et al. (2004), Cayen et al. (2006), Ortega and Rebei (2006), Lubik and Schorfheide (2007) for other monetary policy rule estimations of the Bank of Canada.
normal asymptotics. The estimation results also indicate that the smoothing parameter is highly significant and equal to 0.96. This estimate implies that the Bank of Canada puts forth significant effort to smooth interest rates.

Table 2 illustrates the test statistics for GMM estimation. The Hansen’s $J$-statistic does not reject the null hypothesis that the overidentifying restrictions are satisfied at conventional significance levels.

Despite their significance, one should be wary about GMM-based results that are obtained under the asymptotic normality of the sampling distributions obtained under conventional asymptotics. Under weak-identification asymptotics, the sampling distributions are quite far from being normally distributed. In this chapter, we address the problem of weak identification by using two different tests. The first of these tests is the Anderson and Roubin (1949) test ($AR$-test) in its general form presented by Kleibergen (2002). The second test is the $K$-test developed by Kleibergen (2002). These two tests are robust in the case of nonlinear models (see Stock et al., 2002; Dufour, 2003; Dufour and Taamouti, 2005, 2006), and perhaps more importantly, they are robust even to excluded instruments (see Dufour, 2003). Since it is rarely possible to use all possible instruments, this latter property is quite important from an applied point of view (see Yazgan and Yilmazkuday, 2005).

$AR$ and $K$-test statistics are used to test the null hypothesis that:

$$H_0 : r = 1.37; \chi _\pi = 5.50; \chi _x = 0.09; \rho _i = 0.96$$

i.e. given the instruments that we used, whether the estimated parameters are compatible with the data or not.\textsuperscript{14} Since both of these tests are fully robust to weak instruments (see Stock et al., 2002, pp.522), a non-rejection of this null hypothesis means that our estimates

\textsuperscript{14}As suggested by Kleibergen (2002), the AR-test and the K-test statistics are calculated by interpreting all data matrices in the test as residuals from the projection on exogenous variables.
are also “data-admissible” even under the case of weak instruments.

As is evident from Table 2, given the high p-value of the \(AR\)-test, our parameter estimates cannot be rejected.\(^{15}\) In other words, our GMM estimates of the Bank of Canada’s monetary policy cannot be refuted by the Canadian data.

However, as argued by Kleibergen (2002), the deficiency of the \(AR\)-statistic is that its limiting distribution has a degree of freedom parameter equal to the number of instruments. Therefore the \(AR\)-statistic suffers from the problem of low power when the number of instruments highly exceeds the number of parameters. Kleibergen proposed a statistic (\(K\)-statistic) that remedies the drawback of the \(AR\)-statistic. Kleibergen, unlike the \(AR\)-test, does not provide a finite sample theory, but instead shows that his \(K\)-statistic follows an asymptotic \(\chi^2(G)\) distribution (where \(G\) is the number of endogenous regressors) under the null hypothesis in the absence of exogenous regressors. As can be seen from Table 2, our \(K\)-statistics provides a similar result to the \(AR\)-test.

**Estimation of the New-Keynesian Phillips Curve**

We continue with the structural estimation of the New-Keynesian Phillips curve defined by Equation (II.4) where the expected signs of \(\alpha\) and \(\beta\) are both positive. We use exactly the same methodology that we used for the estimation of the monetary policy rule. The estimation results are illustrated in Table 3. The instruments we use for the GMM estimation consist of six lags of home inflation, six lags of the percentage change in terms of trade and two lags of percentage change in M1. As is evident, both estimates satisfy their expected signs.

\(^{15}\)The \(AR\)-statistics, under the above null hypothesis, has an exact Fisher distribution with \(k\) and \(T-k\) degrees of freedom (where \(k\) is the number of instruments, and \(T\) is the number of observations), given that the error terms are i.i.d. normal, and the instruments are strictly exogenous. \(k \times AR\) statistics are asymptotically distributed chi-square with \(k\) degrees of freedom even without i.i.d. normal errors under standard regularity conditions (see Dufour and Jasiak, 2001, pp. 829, and Dufour 2003, pp.20).
Although the comparison of these estimates with the existing literature is difficult due to the differences in model specifications and sample periods, see Ambler et al. (2004), Murchison et al. (2004), Dufour et al. (2006), Lubik and Schorfheide (2007) for recent New-Keynesian Phillips curve estimations of the Canadian economy. Finally, both AR and \( K \)-statistics in Table 4 support our estimation results for the Phillips curve.

**Remaining Parameters**

Before we continue with our simulation analysis, we set the serial correlation parameters for productivity, trade costs and foreign interest rate as \((\rho_z; \rho_T; \rho_{i^*}) = (0.98; 0.97; 0.99)\) by estimating the relevant AR(1) processes given in the text and the Appendices. Moreover, the related standard deviations, which are used to determine the size of the shocks in our simulations next section, are similarly estimated as \((\sigma_z; \sigma_T; \sigma_{i^*}) = (0.01; 0.09; 0.17)\). We set the share of domestic consumption allocated to imported goods to \(\gamma = 0.36\), which is (implied by Equation (II.34) in the Appendices as) the mean ratio of the value of imports to the value of GDP over the sample period. Finally, we set the gross markup equal to \(\zeta = 1.35\), which is equal to the average markup in the manufacturing sector in Canada, and thus, it is implied that price elasticity of demand faced by each monopolist is set as \(\theta = 3.85\). Now, we have each parameter used in the utility-based welfare function (i.e., Equation (II.9)) and the model solution given in Appendix H. By using our model solution, we start our simulation analysis based on the Canadian economy in the next section.

**Results and Comparisons**

In order to compare the expected loss implications of alternative monetary policy rules, a criterion is needed. We consider two alternative approaches that are highly accepted
in the literature: i) utility-based loss function, ii) *ad hoc* loss function

While the utility-based loss function is obtained through the microfoundations of our model, the *ad hoc* loss function is assumed to depend on the volatility in inflation, the output gap, and the interest rate. We provide the details of each approach in the following subsections.

**Utility-Based Loss Function**

The utility-based loss function implied by Equation (II.9) is as follows:

\[
E_t \sum_{k=0}^{\infty} \beta^{t+k} L_{t+k}^{ub} \tag{II.11}
\]

\[
= E_t \sum_{k=0}^{\infty} \beta^{t+k} \left( \frac{\theta (1 - \log \varsigma - \tau_{t+k}) (\pi_{H,t+k})^2}{2\lambda_w} + \frac{(x_{t+k})^2}{2} - (\log \varsigma + \tau_{t+k} - \gamma) s_{t+k} \right)
\]

The estimated policy function is evaluated relative to the optimal policy as follows:

1. Since a typical central bank determines its policy considering the dynamics of the economy (i.e., the IS curve and the New Keynesian Phillips curve), given these dynamics, following the lead of Ambler et al. (2004), Cayen et al. (2006), Murchison and Rennison (2006), and Ortega and Rebei (2006), we search for the optimized monetary policy rules under possible types of shocks. In particular, we use the method of stochastic simulations to determine the vector of parameters that minimizes the expected loss function; i.e., for each possible combination of \( \rho_t, \chi_\pi, \) and \( \chi_x \) values in Equation (II.8), we calculate the expected loss value by Equation (II.11).

2. We compare the performance of the estimated monetary policy rule of the Bank of Canada with the optimized monetary policy rule (obtained by Exercise 1) in terms of expected loss in the economy (i.e., Equation (II.11)).
In both exercises, we consider a combination of three possible types of shocks, namely a trade cost shock, a technology shock, a foreign interest rate shock. These shocks are determined by Equations (II.17), (II.26) and (II.3). We set the size of the shocks equal to one standard deviation of the relevant shock variables. In other words, we compute the standard deviation of the observed shocks and use them in the simulation.

The results of both exercises are given in Table 5 which compares optimal monetary policy rules and historical (i.e., estimated) monetary policy rules. Note that we have considered the cases of with and without trade cost to show the effect of trade costs. While the case with trade costs refers to the unrestricted version of our model, the case without trade costs refers to the restricted version of our model in which trade costs are ignored (i.e., $\tau_t = 0$ for all $t$). In both cases, optimal $\chi_\pi$ and $\chi_x$ values are much higher than the estimates of historical monetary policy rule of the Bank of Canada. Nevertheless, $\rho_i$ values are very close to each other. In other words, given the utility-based welfare function, the Bank of Canada places much lower weight upon inflation and output than the optimal monetary policy, while it gives approximately the same weight to smoothing the interest rate.

When we compare the welfare loss values calculated by Equation (II.11), we see that the historical monetary policy rule is far from optimal. Moreover, when we compare the discounted (lifetime) value of the deviation of consumption between optimal and historical monetary policy rules, we see that the consumption implied by the historical rule deviates around 50% from the one implied by the optimal rule, in the case with trade costs. This deviation increases to around 90% in the case without trade costs. This brings another possibility into the picture: What if the Bank of Canada has its own expected loss function

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16MATLAB version 7.1.0.246 R(14) Service Pack 3 has been used for the simulation. The codes are available upon request.
rather than the utility-based loss function? We consider this possibility in the following subsection by considering an \textit{ad hoc} loss function.

\textbf{Ad Hoc Loss Function}

Similar to Svensson (1997), Rotemberg and Woodford (1997), Rudebusch and Svensson (1998), Woodford (1999), Batini and Nelson (2001), Smets (2003), the \textit{ad hoc} intertemporal loss function is assumed to depend on the deviations of inflation and output from their steady state values, and the volatility of the policy instrument. It can be demonstrated as follows:

\[ E_t \sum_{k=0}^{\infty} \kappa^k L_{t+k}^{ah} \]  

(II.12)

where \( \kappa \) is the discount factor of the central bank (which can be different from the consumer discount factor, \( \beta \)), and the period loss function, following Smets (2003), is given by:

\[ L_{t}^{ah} = \psi_{\pi} (\pi_{H,t})^2 + (1 - \psi_{\pi}) \left( \psi_{x} (x_{t})^2 + (1 - \psi_{x}) (\Delta i_t)^2 \right) \]  

(II.13)

where \( 0 \leq \psi_{\pi} \leq 1 \) and \( 0 \leq \psi_{x} \leq 1 \). While the inclusion of inflation and output into the loss function is almost standard, as Cayen et al. (2006) point out, the policy instrument may enter as an argument of the loss function for three different reasons: (i) big and unexpected changes to interest rates may cause problems for financial stability (Cukierman 1990; Smets 2003), (ii) the policy-makers may be concerned about hitting the lower nominal bound on interest rates (Rotemberg and Woodford 1997; Woodford 1999; Smets 2003), or (iii) in reality, the monetary authority (and other agents) may be uncertain about the nature and the persistence of the shocks at play in the economy at the time it must make a decision about its policy instrument.
Following Rudebusch and Svensson (1998), we consider the limiting case of the central bank discount factor satisfying $\kappa = 1$ in order to interpret the intertemporal loss function as the unconditional mean of the period loss function, which is equal to the sum of the unconditional variances of the goal variables:

$$E \left[ L_t^{ah} \right] = \psi_\pi \text{Var} \left[ \pi_{H,t} \right] + (1 - \psi_\pi) \left( \psi_x \text{Var} \left[ x_t \right] + (1 - \psi_x) \text{Var} \left[ \Delta i_t \right] \right)$$  \hspace{1cm} (II.14)

Instead of assuming specific values as in the related empirical literature (see Batini and Nelson, 2001; Rudebusch and Svensson, 1998; Cayen et al., 2006), we consider different possible values for $\psi_\pi$ and $\psi_x$ in our analysis. In particular, we employ the following exercises:

1. By considering all possible values for $\psi_\pi$ and $\psi_x$, we analyze the performance of our estimated model (i.e., by using the estimated parameters of the New Keynesian Phillips curve and monetary policy rule) in terms of the expected loss function, after possible types of shocks.

2. Since a typical central bank determines its policy considering the dynamics of the economy (i.e., the IS curve and the New Keynesian Phillips curve), given these dynamics, following the lead of Ambler et al. (2004), Cayen et al. (2006), Murchison and Rennison (2006), and Ortega and Rebei (2006), we search for the optimized monetary policy rules under possible types of shocks, again by considering all possible values for $\psi_\pi$ and $\psi_x$.

3. By considering the expected loss functions calculated by Exercise 1 and Exercise 2, we compare the performance of the estimated monetary policy rule of the Bank of Canada with the optimized monetary policy rule in terms of expected loss in the economy. By this comparison, we search for the weights assigned to inflation, output
and interest rate volatilities in the loss function at which the Bank of Canada is most successful. We follow an optimistic approach and accept these calculated weights as the Bank of Canada’s policy weights.

We depict the details of each exercise in the following subsections. In all exercises, we again consider three possible types of shocks, namely a negative foreign interest rate shock, a negative trade cost shock and a positive technology shock. We again set the size of the shocks equal to one standard deviation of the relevant shock variables.

**Exercise 1**

This subsection calculates the expected loss function given by Equation (II.14) considering the estimated model parameters in Section III (i.e., the estimated parameters of the New Keynesian Phillips curve and monetary policy rule) together with all possible $\psi_\pi$ and $\psi_x$ values. We also consider two cases: one with trade cost, the other without trade costs. The results are given in Figure 1 and Figure 2. As is evident, roughly speaking, the expected loss function decreases in $\psi_\pi$ and increases in $\psi_x$ for Figure 1, while it is slightly different for Figure 2. The intuition behind this result will be clearer by the following exercises.

**Exercise 2**

This subsection searches for the optimized monetary policy rules (MPRs) with and without trade costs. As before, following the lead of Cayen et al. (2006), and Murchison and Rennison (2006), we use the method of stochastic simulations to determine the vector of parameters that minimizes the expected loss function. In particular, for each possible combination of $\rho_i$, $\chi_{\pi}$, $\chi_x$ and $\chi_s$ values in Equation (II.8), we calculate the variance of
inflation, the output gap, and the change in the level of the interest rate to find the mini-
mized expected loss, after simultaneous shocks of technology, trade cost and foreign interest
rate. We again consider all possible \((\psi_\pi, \psi_x)\) pairs in our analysis. Our grid search in the
existence of trade costs results in the expected loss values in Figure 3 which are computed
through Equation (II.8) by using the calculated optimal monetary policy coefficients given
in Figures 4-6.

As is evident from Figure 3, the expected loss function under optimal policy rules
increases in \(\psi_x\) while it takes its lowest value when we move toward \(\psi_\pi = 1\). When we look
at the optimal monetary policy rules under possible \((\psi_\pi, \psi_x)\) pairs in Figures 4-6, we see
that the optimal \(\chi_\pi, \chi_x\) and \(\rho_t\) take higher values when \(\psi_\pi\) decreases.

When we repeat the same analysis in the absence of trade costs, the effects of the
inclusion of trade costs become clearer. The results are given in Figures 7-10.

Figure 3-10 show that the loss function specification of the central bank (i.e., the
\((\psi_\pi, \psi_x)\) values) together with the inclusion of trade costs plays a big role in the determina-
tion of optimized MPRs. We use this information to compare the performance of estimated
MPR with the optimized MPRs in the following exercise.

**Exercise 3**

By considering the expected loss functions calculated by Exercise 1 and Exercise
2, this subsection compares the performance of the estimated (historical) monetary policy
rule of the Bank of Canada with the performance of the optimized monetary policy rule
in terms of expected loss in the economy, under all possible \((\psi_\pi, \psi_x)\) pairs together with
considering the effect of trade costs. By this comparison, we search for the weights assigned
to inflation, output and interest rate volatilities in the loss function by which the actions of
the Bank of Canada are explained best.

In particular, we consider three different cases:

1. The presence of trade costs, i.e., the unrestricted version of our model.

2. The absence of trade costs, i.e., the restricted version of our model in which \( \tau_t = 0 \) for all \( t \).

3. The hybrid case in which trade costs exist, but the Bank of Canada ignores them.

For Case 1, we compare the expected loss values in Figure 1 and Figure 3. We make this comparison by calculating the percentage deviation of the expected loss under estimated monetary policy from the one under optimal monetary policy. The results are given in Figure 11. As is evident from Figure 11, the percentage deviation takes lower values towards \((\psi_\pi, \psi_x) = (0.9, 0.7)\) at which it reaches its minimum. According to these values, for Case 1, it follows that the Bank of Canada assigns 90\% of weight to inflation, 7\% of weight to output gap and 3\% weight to interest rate in the loss function.

According to the calculated weights, the optimal MPR for Case 1 is implied as follows:

\[
\chi^o_\pi = 2.2; \chi^o_x = 0.08; \rho^i_\pi = 0.57
\]

Compared to the estimated/historical MPR in Table 1, the optimal \( \chi^o_\pi = 2.2 \) and \( \rho^i_\pi = 0.57 \) values are lower while the optimal \( \chi^o_x = 0.08 \) value is almost the same.

For Case 2, we compare the expected loss values in Figure 2 and Figure 7. We make this comparison again by calculating the percentage deviation of the expected loss under the estimated monetary policy from the one under optimal monetary policy. The results are given in Figure 12. As is evident from Figure 12, the percentage deviation takes
lower values toward \((\psi_\pi, \psi_x) = (0.1, 0.1)\) at which it reaches its minimum. According to these values, for Case 2, it is implied that the Bank of Canada assigns 10% of weight to inflation, 9% of weight to output gap and 81% weight to interest rate in the loss function.

According to the calculated weights, the optimal MPR for Case 2 is implied as follows:

\[
\chi_\pi^o = 0.9; \chi_x^o = 0.27; \rho_i^o = 0.85
\]

Compared to the estimated/historical MPR in Table 1, the optimal \(\chi_\pi^o = 0.9\) and \(\rho_i^o = 0.85\) values are lower while the optimal \(\chi_x^o = 0.27\); is higher.

For Case 3, we compare the expected loss values in Figure 2 and Figure 3. We again make this comparison by calculating the percentage deviation of the expected loss under the estimated monetary policy from the one under optimal monetary policy. The results are given in Figure 13.

As is evident from Figure 13, the percentage deviation takes lower values toward \((\psi_\pi, \psi_x) = (0.7, 0.5)\) at which it reaches its minimum. According to these values, for Case 3, it is implied that the Bank of Canada assigns 70% of weight to inflation, 15% of weight to output gap and 15% weight to interest rate in the loss function.

According to the calculated weights, the optimal MPR for Case 3 is implied as follows:

\[
\chi_\pi^o = 2.2; \chi_x^o = 0.08; \rho_i^o = 0.57
\]

which is the same as in Case 1.

Now, we have to find a criterion to evaluate which case is more likely to represent the actions of the Bank of Canada. We achieve this by considering the percentage deviation
of the historical monetary policy rule from the optimal monetary policy rule in terms of 
expected loss values for each case. The results are given in Table 6. As is evident, the 
minimum percentage deviation is achieved by the Hybrid Case, which suggests that the 
actions of the Bank of Canada are explained best when trade costs in fact exist but the 
Bank of Canada ignores them.\footnote{17}

**Impulse Response Functions**

This subsection compares the impulse response functions under the estimated (his-
torical) monetary policy with the ones under optimal monetary policy (both utility-based 
and ad hoc), after possible types of shocks. We consider the cases with trade costs in our 
analysis. The results under simultaneous shocks of technology, trade cost and foreign inter-
est rate are given in Figures 14-17. We consider simultaneous shocks rather than individual 
shocks, because, according to our data, they are the possible shocks that the economy can 
experience in a typical period.

Figure 14 compares the response of output gap to three simultaneous shocks under 
estimated and optimal MPRs. As is evident, the volatility in output gap is best controlled 
under estimated MPR, while it is highest under optimized MPR found by the ad hoc 
expected loss function. Nevertheless, it is the opposite case for inflation when we consider 
Figure 15: the volatility in inflation is best controlled under optimized MPR found by the ad 
hoc expected loss function, while it is highest under estimated MPR. Similar comparisons 
can be made in Figures 16-17.

\footnote{17When we compare the discounted (lifetime) value of the deviation of consumption between optimal and historical monetary policy rules, we see that the consumption implied by the historical rule deviates around 101% from the one implied by the optimal rule, in the presence of trade costs. The deviation is around 18% in the absence of trade costs. In the Hybrid Case, the deviation is calculated as 99%.}
Conclusions

We introduced an open economy DSGE model to analyze the effects of trade costs on the actual central bank behaviour. The log-linearized model is expressed in terms of four blocks of equations: aggregate demand (i.e., the IS curve), aggregate supply (i.e., the New-Keynesian Phillips curve), monetary policy rule, and stochastic processes. We estimated the New-Keynesian Phillips curve for the Canadian economy together with the monetary policy rule of the Bank of Canada.

By considering the dynamics of the Canadian economy (i.e., the New-Keynesian Phillips curve and the IS curve), we calculated optimal monetary policy rules under different scenarios and compared them with the estimated monetary policy rule to have a better insight for the actions of the Bank of Canada. When we consider a utility-based expected loss function, we find that the actions of the Bank of Canada are far from being optimal. When we consider an ad hoc expected loss function based on inflation, output and interest volatilities, we find the actions of the Bank of Canada are best explained by a model in which trade costs actually exist in the economy but the Bank of Canada ignores them. Finally, we find that the Bank of Canada assigns 70% of weight to inflation, 15% of weight to interest rate and 15% of weight to output in its ad hoc loss function.

Many things remain to be done, in terms of either modeling or empirical analysis: what if trade costs affect both final good and intermediate input prices; what is the relation between capital (utilization) and trade costs (and/or oil prices); is there any difference in terms of the trade cost effects between the monetary policy of developing and developed countries (e.g., small versus large economies)? These are possible topics of future research.
Table 1. GMM Estimates of the Monetary Policy Rule

<table>
<thead>
<tr>
<th></th>
<th>$r$</th>
<th>$\chi_\pi$</th>
<th>$\chi_x$</th>
<th>$\rho_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.37</td>
<td>5.50</td>
<td>0.09</td>
<td>0.96</td>
</tr>
<tr>
<td></td>
<td>(0.8441)</td>
<td>(4.1913)</td>
<td>(0.0616)</td>
<td>(0.0258)</td>
</tr>
<tr>
<td></td>
<td>[0.0523]</td>
<td>[0.0946]</td>
<td>[0.0835]</td>
<td>[0.0000]</td>
</tr>
</tbody>
</table>

Notes: Standard errors calculated using the Delta method are in parentheses and p-values are in brackets. The sample size is 114 after considering data availability and instruments used which consist of twelve lags of home inflation, percentage change in M1 and three lags of output gap.
Table 2. Test Statistics for GMM Estimation of the Monetary Policy Rule

<table>
<thead>
<tr>
<th>$AR$–stat</th>
<th>$K$–stat</th>
<th>$J$–stat</th>
<th>Adjusted $R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F(27; 87)$</td>
<td>$\chi^2 (27)$</td>
<td>$\chi^2 (2)$</td>
<td>$\chi^2 (25)$</td>
</tr>
<tr>
<td>0.74</td>
<td>19.87</td>
<td>2.39</td>
<td>15.53</td>
</tr>
<tr>
<td>[0.82]</td>
<td>[0.84]</td>
<td>[0.30]</td>
<td>[0.93]</td>
</tr>
</tbody>
</table>

Notes: P-values are in brackets.
Table 3. GMM Estimates of the New Keynesian Phillips Curve

<table>
<thead>
<tr>
<th></th>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$\lambda_x$</th>
<th>$\lambda_m$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.99</td>
<td>0.99</td>
<td>1.09</td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td>(0.02)</td>
<td>(0.01)</td>
<td>(0.00)</td>
<td>(0.00)</td>
</tr>
<tr>
<td></td>
<td>[0.00]</td>
<td>[0.00]</td>
<td>[0.00]</td>
<td>[0.00]</td>
</tr>
</tbody>
</table>

Notes: Standard errors are in parentheses and p-values are in brackets. The sample size is 127 after considering data availability and instruments used, which consist of six lags of home inflation, six lags of the percentage change in terms of trade and two lags of percentage change in M1. The standard errors of $\lambda_x$ and $\lambda_m$ have been calculated by using the Delta method.
Table 4. Statistics for GMM Estimation of the New Keynesian Phillips Curve

<table>
<thead>
<tr>
<th>$AR_{-}$stat</th>
<th>$K_{-}$stat</th>
<th>$J_{-}$stat</th>
<th>Adjusted $R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F(14;113)$</td>
<td>$\chi^2(14)$</td>
<td>$\chi^2(2)$</td>
<td>$\chi^2(13)$</td>
</tr>
<tr>
<td>0.65</td>
<td>9.03</td>
<td>0.05</td>
<td>11.99</td>
</tr>
<tr>
<td>[0.82]</td>
<td>[0.83]</td>
<td>[0.98]</td>
<td>[0.98]</td>
</tr>
</tbody>
</table>

Notes: P-values are in brackets.
Table 5. Optimal vs. Historical Monetary Policy Rule

<table>
<thead>
<tr>
<th>Policy Type</th>
<th>$\chi_\pi$</th>
<th>$\chi_x$</th>
<th>$\rho_i$</th>
<th>Welfare Loss</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimal MPR with Trade Costs</td>
<td>18.5</td>
<td>0.37</td>
<td>0.97</td>
<td>$1.11 \times 10^{-5}$</td>
</tr>
<tr>
<td>Optimal MPR without Trade Costs</td>
<td>13.0</td>
<td>0.36</td>
<td>0.95</td>
<td>$2.29 \times 10^{-5}$</td>
</tr>
<tr>
<td>Historical MPR with Trade Costs</td>
<td>5.5</td>
<td>0.09</td>
<td>0.96</td>
<td>12.76</td>
</tr>
<tr>
<td>Historical MPR without Trade Costs</td>
<td>5.5</td>
<td>0.09</td>
<td>0.96</td>
<td>13.65</td>
</tr>
</tbody>
</table>
Table 6. Expected Loss Values

<table>
<thead>
<tr>
<th>Case</th>
<th>Monetary Policy Rule</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Estimated MPR</td>
<td>Optimized MPR</td>
</tr>
<tr>
<td>Presence of Trade Costs</td>
<td>$3.77 \times 10^{-6}$</td>
<td>$3.44 \times 10^{-6}$</td>
</tr>
<tr>
<td>Absence of Trade Costs</td>
<td>$1.60 \times 10^{-6}$</td>
<td>$2.34 \times 10^{-8}$</td>
</tr>
<tr>
<td>Hybrid Case</td>
<td>$4.40 \times 10^{-6}$</td>
<td>$4.40 \times 10^{-6}$</td>
</tr>
</tbody>
</table>

Notes: MPR stands for Monetary Policy Rule. Percentage deviation is defined as 100 times the log difference between the expected loss functions under estimated MPR and optimized MPR.
Figure 1. Expected Loss Values for Historical MPR in the presence of Trade Costs

Figure 2. Expected Loss Values for Historical MPR in the absence of Trade Costs
Figure 3. Expected Loss Values for Optimal MPR in the presence of Trade Costs

Figure 4. Optimal Coefficient of Inflation in the presence of Trade Costs

Figure 5. Optimal Coefficient of Output in the presence of Trade Costs
Figure 6. Optimal Coefficient of Interest Rate in the presence of Trade Costs

Figure 7. Expected Loss Values for Optimal MPR in the absence of Trade Costs

Figure 8. Optimal Coefficient of Inflation in the absence of Trade Costs
Figure 9. Optimal Coefficient of Output in the absence of Trade Costs

Figure 10. Optimal Coefficient of Interest Rate in the presence of Trade Costs

Figure 11. Percentage Deviation from Optimal Expected Loss in the Presence of Trade Costs
Figure 12. Percentage Deviation from Optimal Expected Loss in the Absence of Trade Costs

Figure 13. Percentage Deviation from Optimal Expected Loss for the Hybrid Case

Figure 14. Response of Output Gap
Figure 15. Response of Inflation

Figure 16. Response of Nominal Interest

Figure 17. Response of Real Exchange Rate
Appendices

This section depicts the microfoundations and the technical details of the model. The log-linearized model is expressed in terms of four blocks of equations: aggregate demand (i.e., the IS curve), aggregate supply (i.e., the New-Keynesian Phillips curve), a monetary policy rule, and stochastic processes. Lower case letters denote log variables; the subscripts $H$ and $F$ stand for domestically produced and imported variables, respectively; the superscript * stands for the variables of the rest of the world; and lastly, a bar on a variable (→) stands for the target.\(^\text{18}\)

A. Definitions and Some Identities

We define the CPI as follows:

$$p_t = (1 - \gamma)p_{H,t} + \gamma p_{F,t}$$  \(\text{(II.15)}\)

where $p_{H,t}$ is the (log) price index for domestically consumed home goods; $p_{F,t}$ is the (log) price index for imported goods; and $\gamma$ is the share of domestic consumption allocated to imported goods. In other words, $\gamma$ represents a natural index of openness. Both $p_{H,t}$ and $p_{F,t}$ are in domestic currency. The price index for imported goods is given by:

$$p_{F,t} = e_t + p_{F,t}^* + \tau_t$$  \(\text{(II.16)}\)

where $e_t$ is the (log) nominal effective exchange rate; $p_{F,t}^*$ is the (log) price index for domestically consumed foreign goods at the source; and $\tau_t$ is the (log) gross trade cost, which is an income received by the rest of the world.\(^\text{19}\) Since we assume that the transportation\(^\text{18}\)Although some of the equations in this section are repeated from Gali and Monacelli (2005) or Yilmazkuday (2007) for the convenience of the reader, the equations with trade costs are new to this paper.\(^\text{19}\)For future reference, $p_{H,t}$ is the (log) price index for the imported goods for the rest of the world, and $p_{F,t}$ is the (log) domestic price index for the rest of the world. We assume that the trade costs consist of transportation costs and transportation sector is owned by the rest of the world, so there is no transporta-
costs are the same across goods and they are symmetric, the (log) gross trade cost directly enters the price index for imported goods. The autoregressive parameter, $\rho_\tau$, appears in the evolution of trade costs as follows:

$$
\tau_t = \rho_\tau \tau_{t-1} + \varepsilon_t^T
$$

(II.17)

where $\rho_\tau \in [0, 1]$ and $\varepsilon_t^T$ is assumed to be an independent and identically distributed (i.i.d.) shock with zero mean and variance $\sigma_\tau^2$.\(^{20}\)

If we define the (log) effective terms of trade as $s_t \equiv p_{F,t} - p_{H,t}$, we can write the CPI formula as:

$$
p_t \equiv (1 - \gamma)p_{H,t} + \gamma p_{F,t}
$$

(II.18)

Thus, we can write the formula of CPI inflation as follows:

$$
\pi_t = \pi_{H,t} + \gamma (s_t - s_{t-1})
$$

(II.19)

where $\pi_{H,t} = p_{H,t} - p_{H,t-1}$ is the inflation of home-produced goods.

By combining $s_t \equiv p_{F,t} - p_{H,t}$ and $p_{F,t} = e_t + p_{F,t}^* + \tau_t$, we can write:

$$
s_t \equiv e_t + p_{F,t}^* + \tau_t - p_{H,t}
$$

(II.20)

We can define the (log) effective real exchange rate as:

$$
q_t = e_t + p_{t}^* - p_t
$$

(II.21)

By using Equations (II.15), (II.16) and (II.20), together with the symmetric versions of inflation income received by the home country. This assumption is reasonable after considering the fact that we are analyzing the inflation targeting experience of Canada after the introduction of NAFTA. Another interpretation of this assumption would be to have iceberg trade costs. See Anderson and van Wincoop (2003) for a discussion of iceberg melt structure of economic geography literature and trade costs.

\(^{20}\)The introduction of an AR(1) process for the trade costs is essential in our simulations below.
Equations (II.15) and (II.16) for the rest of the world, we can rewrite Equation (II.21) as follows:

$$q_t = (1 - \gamma - \gamma^*)s_t - (1 - 2\gamma^*)\tau_t$$  \hspace{1cm} (II.22)

where $\gamma^*$ is the share of foreign consumption allocated to goods imported from the home country. In a special case in which the home country is a small one (i.e., $\gamma^*$ is a very small number), Equation (II.22) can be approximated as:

$$q_t \approx (1 - \gamma)s_t - \tau_t$$  \hspace{1cm} (II.23)

Compared to the studies in the literature that ignore trade costs in open economy models, such as Parrado (2004), Gali and Monacelli (2005), Lubik and Schorfheide (2007), and Yilmazkuday (2007), the presence of trade costs is important in Equations (II.22) and (II.23). In particular, as is shown empirically by Caves et al. (1990), Crucini et al. (2005), Engel (1983), Engel and Rogers (1996), Krugman and Obstfeld (1991), Lutz (2004), Parsley and Wei (2000), Rogers and Jenkins (1995), trade costs play a big role in the determination of real exchange rates.

The uncovered interest parity condition is given by:

$$i_t = i_t^* + E_t [e_{t+1}] - e_t$$  \hspace{1cm} (II.24)

where $E$ is the expectation operator. The derivation of this condition is given in Appendix B. Equation (II.24) relates the movements of the interest rate differentials to the expected variations in the effective nominal exchange rate. Since $s_t \equiv e_t + p_{F,t}^* - p_{H,t}$, we can rewrite Equation (II.24) as follows:

$$s_t = (i_t^* - E_t [\pi_{F,t+1}^*]) - (i_t - E_t [\pi_{H,t+1}]) + E_t [s_{t+1} - \Delta\tau_{t+1}]$$  \hspace{1cm} (II.25)
where $\Delta \tau_{t+1}$ is the change in trade cost from period $t$ to $t+1$. Equation (II.25) shows the terms of trade between the home country and the rest of the world as a function of current interest rate differentials, expected future home inflation differentials and its own expectation for the next period together with the expected future change in trade cost. Here, the evolution of foreign interest rate shock is given by:

$$i^*_t = \rho_{i^*} i^*_{t-1} + \varepsilon^*_t$$  \hspace{1cm} (II.26)

where $\rho_{i^*} \in [0, 1]$, and $\varepsilon^*_t$ is assumed to be an independent and identically distributed (i.i.d.) shock with zero mean and variance $\sigma_{i^*}^2$.

### B. Individuals

We can make our analysis for a representative individual who has the following intertemporal lifetime utility function:

$$E_t \left[ \sum_{k=0}^{\infty} \beta^k \{ U(C_{t+k}) - V(N_{t+k}) \} \right]$$  \hspace{1cm} (II.27)

where $0 < \beta < 1$ is the discount factor; $U(C_t)$ is the utility out of consuming a composite index of $C_t$; and $V(N_t)$ is the disutility out of working $N_t$ hours. The composite consumption index, $C_t$, is defined by:

$$C_t = (C_{H,t})^{1-\gamma}(C_{F,t})^\gamma$$  \hspace{1cm} (II.28)

Consumption sub-indexes, $C_{H,t}$ and $C_{F,t}$, are symmetric. These Dixit-Stiglitz type indices are defined by:

$$C_{H,t} = \left[ \int_0^1 C_{H,t}(j)^{(\theta-1)/\theta} dj \right]^{\theta/(\theta-1)} \hspace{1cm} \text{and} \hspace{1cm} C_{F,t} = \left[ \int_0^1 C_{F,t}(j)^{(\theta-1)/\theta} dj \right]^{\theta/(\theta-1)}$$  \hspace{1cm} (II.29)
where \( \theta > 1 \) is the price elasticity of demand faced by each monopolist and \( C_{H,t}(j) \) and \( C_{F,t}(j) \) are the quantities purchased by home agents of domestic and imported goods, respectively. The optimality conditions result in:

\[
C_{H,t}(j) = \left[ \frac{P_{H,t}(j)}{P_{H,t}} \right]^{-\theta} C_{H,t} \tag{II.30}
\]

\[
C_{F,t}(j) = \left[ \frac{P_{F,t}(j)}{P_{F,t}} \right]^{-\theta} C_{F,t}
\]

where

\[
P_{H,t} = \left[ \int_0^1 ([P_{H,t}(j)])^{1-\theta} \, dj \right]^{1/(1-\theta)} \tag{II.31}
\]

and

\[
P_{F,t} = \left[ \int_0^1 ([P_{F,t}(j)])^{1-\theta} \, dj \right]^{1/(1-\theta)} \tag{II.32}
\]

Similarly, the demand allocation for home and imported goods implies:

\[
C_{H,t} = \frac{(1-\gamma) C_t P_t}{P_{H,t}} \tag{II.33}
\]

and

\[
C_{F,t} = \frac{\gamma P_t C_t}{P_{F,t}} \tag{II.34}
\]

where \( P_t = \left( P_{H,t} \right)^{1-\gamma} \left( P_{F,t} \right)^{\gamma} \) is the consumer price index (CPI).

The individual household constraint is given by:

\[
\int_0^1 [P_{H,t}(j)C_{H,t}(j) + P_{F,t}(j)C_{F,t}(j)] \, dj + E_t [F_{t,t+1}B_{t+1}] = W_t N_t + B_t + T_t \tag{II.35}
\]
where $F_{t,t+1}$ is the stochastic discount factor, $B_{t+1}$ is the nominal payoff in period $t + 1$ of the portfolio held at the end of period $t$, $W_t$ is the hourly wage, and $T_t$ is the lump sum transfers/taxes.

By using the optimal demand functions, we can rewrite (II.35) in terms of the composite good as follows:

$$P_tC_t + E_t[F_{t,t+1}B_{t+1}] = W_tN_t + B_t + T_t \quad \text{(II.36)}$$

The home agent’s problem is to choose paths for consumption, portfolio and the output of good $j$. Therefore, the representative consumer maximizes her expected utility [equation (II.27)] subject to the budget constraint [equation (II.36)]. By FOC, we obtain:

$$\beta E_t \left[ \frac{U_C(C_{t+1}) P_t}{U_C(C_t) P_{t+1}} \right] = \frac{1}{I_t} \quad \text{(II.37)}$$

where $I_t = 1/E_t [F_{t,t+1}]$ is the gross return on the portfolio. Equation (II.37) represents the traditional intertemporal Euler equation for total real consumption. We also obtain the labor supply decision of the individual as follows:

$$\frac{W_t}{P_t} = \frac{V_N(N_t)}{U_C(C_t)} \quad \text{(II.38)}$$

The problem is analogous for the rest of the world. The Euler equation for the rest of the world would thus be:

$$\beta E_t \left[ \frac{u_C^*(C_{t+1}) P_t^* \Xi_t}{u_C^*(C_t^*) P_{t+1}^* \Xi_{t+1}} \right] = E_t [F_{t,t+1}] \quad \text{(II.39)}$$

where $\Xi_t$ is the nominal effective exchange rate. By combining Equations (II.37) and (II.39), together with assuming $U(C) = \log C$, one can obtain:

$$C_t = C_t^* Q_t \quad \text{(II.40)}$$
for all $t$, where $Q_t = \Xi_t P_t^*/P_t$ is the real effective exchange rate. Under the assumption of complete international financial markets, by combining log-linearized version of Equations (II.37), (II.39) and (II.40) together with Equation (II.21) (the log linear version of $Q_t = \Xi_t P_t^*/P_t$), one can obtain:

$$i_t = i_t^* + E_t [e_{t+1}] - e_t$$  \hspace{1cm} (II.41)

where $i_t = \log (I_t) = \log (1/ (E_t [F_{t,t+1}])))$ and $i_t^* = \log (\Xi_t / (E_t [F_{t,t+1} \Xi_{t+1}]))$. Equation (II.41) is the uncovered interest parity condition given by Equation (II.24) in the text.

After introducing the micro-foundations of aggregate demand, we can now find a log-linearized equation for the IS curve. From now on, lower case variables will denote the log variables, and the capital letters without time subscript will denote steady-state values of the respective ratios.

**C. Firms**

We assume that the production function is as follows:

$$Y_t (j) = Z_t N_t (j)$$  \hspace{1cm} (II.42)

where $Z_t$ is an exogenous economy-wide productivity parameter; and $N_t$ is labor input. Accordingly, the marginal cost of production is given by:

$$MC_t^m = (1 - \omega) \frac{W_t}{Z_t}$$  \hspace{1cm} (II.43)
where $\omega$ is the employment subsidy. By also using Equation (II.38) together with assuming $U(C) = \log C$ and $V(N) = N$, we can write the real marginal cost as follows:\footnote{Balanced growth requires the relative risk aversion in consumption to be unity, and thus we set $U(C) = \log C$. Following the lead of Hansen (1985), we also assume that labor is indivisible, implying that the representative agent’s utility is linear in labor hours so that $V(N) = N$.}

$$mc_t = \log (1 - \omega) + w_t - p_{H,t} - z_t \quad (\text{II.44})$$

Moreover, if we define the aggregate output in the home country as

$$Y_t = \left[ \int_0^1 Y_t(j)^{(\theta-1)/\theta} dj \right]^{\theta/(\theta-1)}$$

labor market equilibrium implies:

$$N_t = \int_0^1 N_t(j) dj \frac{Y_t A_t}{Z_t} \quad (\text{II.45})$$

where $A_t = \int_0^1 Y_t(j) Y_t(j) \frac{Y_t(j)}{Z_t} dj$ of which equilibrium variations can be shown to be of second order in log terms. Thus, we can write:

$$y_t = z_t + n_t \quad (\text{II.46})$$

**D. Aggregate Demand**

For all differentiated goods, market clearing implies:

$$Y_t(j) = C_{H,t}(j) + C^*_H(j) \quad (\text{II.47})$$

By using Equation (II.30), we can rewrite it as follows:

$$Y_t(j) = \left[ \frac{P_{H,t}(j)}{P_{H,t}} \right]^{-\theta} C^A_{H,t} \quad (\text{II.48})$$
where $C_{H,t} = C_{H,t} + C_{H,t}^*$ is the aggregate world demand for the goods produced in the home country. By using Equation (II.33) and the symmetric version of Equation (II.34) for the rest of the world, we can rewrite it as follows:

$$Y_t(j) = \left( \frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\theta} \left( 1 - \gamma \right) \frac{P_t C_t}{P_{H,t}} + \gamma^* \frac{P_t^* C_t^*}{P_{H,t}^*} \right)$$  \hspace{1cm} (II.49)

By using $Y_t = \left[ \int_0^1 Y_t(j)(\theta-1)/\theta \, dj \right]^{\theta/(\theta-1)}$, we can write:

$$Y_t = \left( 1 - \gamma \right) \frac{P_t C_t}{P_{H,t}} + \gamma^* \frac{P_t^* C_t^*}{P_{H,t}^*}$$  \hspace{1cm} (II.50)

which implies that we can rewrite Equation (II.49) as follows:

$$Y_t(j) = \left[ \frac{P_{H,t}(j)}{P_{H,t}} \right]^{-\theta} Y_t \hspace{1cm} (II.51)$$

Log-linearizing Equation (II.50) around the steady-state together with using $s_t = p_{F,t} - p_{H,t}$ and Equation (II.22) will transform it to the following expression:

$$y_t = c_t + \gamma s_t - \tau_t \hspace{1cm} (II.52)$$

By also using Equation (II.19) and the log-linearized version of Equation (II.37) (i.e., Euler), we can rewrite Equation (II.52) as follows:

$$y_t = E_t (y_{t+1}) - (i_t - E_t (\pi_{H,t+1})) + E_t (\Delta \tau_{t+1}) \hspace{1cm} (II.53)$$

E. The New-Keynesian Phillips Curve

Now, we have the equation of aggregate demand. In order to find the equation of aggregate supply, we have to analyze the producer part. Our derivation draws on Gali and Monacelli (2005) except for the fact that we consider the target inflation as the steady-state.
level of inflation. The model employs a Calvo price-setting process, in which producers are able to change their prices only with some probability, independently of other producers and the time elapsed since the last adjustment. It is assumed that producers behave as monopolistic competitors. Accordingly, each producer faces the following demand function:

\[ Y_t(j) = \left[ \frac{P_{H,t}(j)}{P_{H,t}} \right]^{-\theta} C_{H,t}^A, \]  

where \( C_{H,t}^A = C_{H,t} + C_{H,t}^n \) is the aggregate world demand for the goods produced. Note that this expression is the same with Equation (II.48).

Assuming that each producer is free to set a new price at period \( t \), it follows that the objective function can be written as:

\[
\max_{P_{H,t}} E_t \left[ \sum_{k=0}^{\infty} \alpha^k F_{t,t+k} \left\{ Y_{t+k} \left( \bar{P}_{H,t} - MC_{t+k}^n \right) \right\} \right]  
\]

(II.55)

where \( \bar{P}_{H,t} \) is the new price chosen in period \( t \), and \( \alpha \) is the probability that producers maintain the same price of the previous period. The problem of producers is to maximize equation (II.55) subject to Equation (II.54). The first order necessary condition (FONC) of the firm for this maximization is:

\[
E_t \left[ \sum_{k=0}^{\infty} \alpha^k F_{t,t+k} \left\{ Y_{t+k} \left( \bar{P}_{H,t} - \zeta MC_{t+k}^n \right) \right\} \right] = 0  
\]

(II.56)

where \( \zeta = \theta/(\theta - 1) \) is a markup as a result of market power. Using Equation (II.37), we can rewrite Equation (II.56) as follows:

\[
E_t \left[ \sum_{k=0}^{\infty} (\beta \alpha)^k F_{t,t+k} \left\{ Y_{t+k} \left( \frac{\bar{P}_{H,t}}{P_{H,t-1}} - \zeta \Pi_{t-1,t+k} MC_{t+k}^n \right) \right\} \right] = 0  
\]

(II.57)

where \( \Pi_{t-1,t+k} = \frac{P_{H,t+k}}{P_{H,t}} \) and \( MC_{t+k}^n = \frac{MC_{t+k}}{P_{H,t+k}}. \) Log-linearizing equation Equation (II.57)


54
around trend inflation $\bar{\Pi}$ together with balanced trade results in:

$$\bar{p}_{H,t} = \varphi + p_{H,t-1} + \bar{\Pi} E_t \left[ \sum_{k=0}^{\infty} (\beta \alpha)^k \pi_{H,t+k} \right] + \bar{\Pi} (1 - \beta \alpha) E_t \left[ \sum_{k=0}^{\infty} (\beta \alpha)^k \hat{mc}_{t+k} \right]$$  (II.58)

where $\varphi = 1 - \bar{\Pi} (1 - \bar{\pi})$ and $\bar{\pi} = \log \bar{\Pi}$ are constants; $\hat{mc}_t = mc_t - mc$ is the log deviation of real marginal cost from its steady state value, $mc = -\log \zeta$. Equation (II.58) can be rewritten as:

$$\bar{p}_{H,t} - p_{H,t-1} = (1 - \beta \alpha) \varphi + \beta \alpha E_t [\bar{p}_{H,t} - p_{H,t-1}] + \bar{\Pi} \pi_{H,t} + \bar{\Pi} (1 - \beta \alpha) \hat{mc}_t$$  (II.59)

In equilibrium, each producer that chooses a new price in period $t$ will choose the same price and the same level of output. Then the (aggregate) price of domestic goods will obey:

$$P_{H,t} = \left[ \alpha P_{H,t-1}^{1-\theta} + (1 - \alpha) \bar{P}_{H,t}^{1-\theta} \right]^{1/(1-\theta)}$$  (II.60)

which can be log-linearized as follows:

$$\pi_{H,t} = (1 - \theta) \left( \bar{p}_{H,t} - p_{H,t-1} \right)$$  (II.61)

Finally, by combining Equations (II.59) and (II.61), we obtain an expression for the New-Keynesian Phillips curve (Equation (II.4) in the main text):

$$\pi_{H,t} = \lambda_\pi E_t [\pi_{H,t+1}] + \lambda_m (\varphi + \hat{mc}_t)$$  (II.62)

where $\lambda_\pi = \frac{\beta \alpha}{1 - (1 - \alpha)(\Pi)}$, $\lambda_m = \frac{(1 - \alpha)(1 - \alpha \beta)}{1 - (1 - \alpha)(\Pi)}$, and $\varphi = 1 - \bar{\Pi} (1 - \bar{\pi})$. Note that this expression reduces to zero-inflation steady state New-Keynesian Phillips curve when $\bar{\pi} = 0$ (i.e., $\bar{\Pi} = 1$).
F. Equilibrium Dynamics

We start with obtaining an expression for real marginal cost in terms of output. In particular, we can combine Equations (II.44) and (II.52) as follows:

\[ mc_t = \log (1 - \omega) + y_t - z_t + \tau_t \] (II.63)

By using the symmetric version of Equation (II.52) for the rest of the world, namely \( y_t^* = c_t^* + \gamma^* s_t^* - \tau_t \), together with Equations (II.22) and (II.40), we can also obtain:

\[ y_t = y_t^* + s_t - \tau_t \] (II.64)

As discussed in Rotemberg and Woodford (1999), under the assumption of a constant employment subsidy \( \omega \) that neutralizes the distortion associated with firms’ market power, it can be shown that the optimal monetary policy is the one that replicates the flexible price equilibrium allocation in a closed economy. That policy requires that real marginal costs (and thus mark-ups) are stabilized at their steady state level, which in turn implies that domestic prices be fully stabilized. However, as shown by Gali and Monacelli (2005), there in additional source of distortion in open economy models: the possibility of influencing the terms of trade in a way beneficial to domestic consumers. Nevertheless, an employment subsidy can be found that exactly offsets the combined effects of market power and the terms of trade distortions, thus rendering the flexible price equilibrium allocation optimal. In order to show this, consider the optimal allocation from the social planner’s point of view: maximize Equation (II.27) subject to Equations (II.42), (II.45), (II.50) and (II.51). This optimization results in a constant level of employment, \( N_t = 1 \).

On the other hand, as in Gali and Monacelli (2005), flexible price equilibrium...
satisfies:
\[
\frac{\theta - 1}{\theta} = MC_t \tag{II.65}
\]
where \(MC_t\) stands for real marginal cost at flexible price equilibrium. If we combine Equations (II.38), (II.43), (II.65) with the optimal allocation of the social planner's problem (i.e., \(N_t = 1\)), we can obtain:
\[
\frac{\theta - 1}{\theta} = 1 - \omega \tag{II.66}
\]
which suggests that an employment subsidy can be found that exactly offsets the combined effects of market power and the terms of trade distortions.

After defining domestic natural level of output as the one satisfying flexible price equilibrium (i.e., Equation (II.63) with \(mc_t = -\log \zeta\)), we can write it as follows:
\[
\bar{y}_t = -\log \zeta - \log (1 - \omega) + z_t - \tau_t \tag{II.67}
\]
which can be rewritten by using Equation (II.66) as follows:
\[
\bar{y}_t = z_t - \tau_t \tag{II.68}
\]

Now, we can define output gap as the deviation of (log) domestic output (i.e., \(y_t\)) from the domestic natural level of output as follows:
\[
x_t = y_t - \bar{y}_t \tag{II.69}
\]
By using Equation (II.63), we can also write the (log) deviation of real marginal cost from its steady state in terms of output gap as \(\hat{mc}_t = x_t\), which implies that the New-Keynesian Phillips curve can be written in terms of output gap as follows:
\[
\pi_{H,t} = \lambda_{\pi} E_t [\pi_{H,t+1}] + \lambda_m (\varphi + x_t) \tag{II.70}
\]
By using Equations (II.53), (II.67) and (II.69), we can write the IS curve in terms of output gap as follows:

\[ x_t = E_t (x_{t+1}) - (i_t - E_t (\pi_{H,t+1})) + \Delta z_{t+1} \]  

(II.71)

\[ x_t = E_t (x_{t+1}) - (i_t - E_t (\pi_{H,t+1})) + \Delta z_{t+1} \]  

G. Utility-Based Welfare

The period specific utility from consumption, \( U(C_t) \), and disutility from working, \( V(N_t) \), can be second-order approximated around their steady states as follows:

\[ U(C_t) = c_t + t.i.p. + o (\|a^3\|) \]  

(II.72)

and

\[ V(N_t) = n_t + \frac{1}{2} n_t^2 + t.i.p. + o (\|a^3\|) \]  

(II.73)

where \( t.i.p. \) represents terms independent of policy and \( o (\|a^3\|) \) represents terms that are higher than 3rd order. We have used the steady state relation \( V_N(N) N = U_C(C) C \) together with our assumptions \( U(C) = \log C \) and \( V(N) = N \) for Equations (II.72) and (II.73). By using Equation (II.52), its symmetric version for the rest of the world, \( s_t + s_t^* = 2 \tau_t \), log version of Equation (II.40) and Equation (II.22), we can write the following expression for \( c_t \):

\[ c_t = (1 - \gamma)y_t + \gamma y_t^* + (1 - \gamma)\tau_t \]  

(II.74)

Defining \( \tilde{c}_t = c_t - \bar{c}_t \) as the deviation of (log) consumption from its flexible pricing equilibrium, we can write:

\[ c_t = (1 - \gamma)x_t + (1 - \gamma)\bar{y}_t + \gamma \bar{y}_t^* + (1 - \gamma)\bar{\tau}_t \]  

(II.75)
which can be inserted into Equation (II.72). Related to Equation (II.73), after defining
\( \bar{n}_t = n_t - \bar{n}_t \) as the deviation of (log) employment from its flexible pricing equilibrium, by
using the log version of Equation (II.45), we can write:

\[
n_t = x_t + a_t + \bar{y}_t - \bar{z}_t \tag{II.76}
\]

where \( a_t = \log \left( \int_0^1 \frac{Y_i(j) d\bar{y}}{Y_t} \right) = \log \left( \int_0^1 \frac{P_{H,t}(j) d\bar{j}}{P_{H,t}} \right) \) by using Equation (II.51) and we have used \( \bar{a}_t = 0 \) which is implied by the definition of flexible pricing. Then, by using Equations
(II.75) and (II.76), we can write:

\[
U(C_t) - V(N_t) = - \left( \gamma x_t + a_t + \frac{1}{2} (x_t + a_t + \bar{y}_t - \bar{z}_t)^2 \right) + t.i.p. \tag{II.77}
\]

The following lemmas are helpful for our analysis.

Lemma 1. \( a_t = \frac{9}{2} \text{var}_t (p_{H,t}(i)) + o(\|a^3\|) \).


Lemma 2. \( \sum_{t=0}^{\infty} \beta^t \text{var}_t (p_{H,t}(i)) = \frac{1}{\lambda_w} \sum_{t=0}^{\infty} \beta^t \pi_{H,t}^2 \) where \( \lambda_w = \frac{(1-\alpha)(1-\alpha\beta)}{\alpha} \).

Proof: See Woodford (2003), Chapter 6.

According to our lemmas and Equations (II.26), (II.64), (II.69), (II.77), we can
calculate the utility-based welfare function as follows:

\[
E_t \sum_{k=0}^{\infty} \beta^{t+k} (U(C_{t+k}) - V(N_{t+k})) = E_t \sum_{k=0}^{\infty} \beta^{t+k} (\log \zeta + \tau_{t+k} - \gamma) s_{t+k}
\]

\[
- \frac{1}{2} E_t \sum_{k=0}^{\infty} \beta^{t+k} \left( 1 - \log \zeta - \tau_{t+k} \right) \left( \frac{6}{\lambda_w} (\pi_{H,t+k})^2 + (x_{t+k})^2 \right)
\]

\[
+ t.i.p + o(\|a^3\|) \tag{II.78}
\]

Note that the utility-based welfare function depends on the volatility in inflation and output
gap as well as the trade costs and the terms of trade.
H. Model Solution

The dynamic system is given by the main Equations (II.19), (II.25), (II.2), (II.8), (II.70), by the exogenous shock Equations (II.17), (II.26), (II.3), by the definition of domestic inflation \( \pi_{H,t} = p_{H,t} - p_{H,t-1} \) and by the definition of output gap \( x_t = y_t - \bar{y}_t \) and Equation (II.67). For simplicity, after substituting \( x_t = y_t - \bar{y}_t \) and Equation (II.67) into Equations (II.2), (II.8), (II.70) and after substituting Equation (II.19) into Equation (II.8), we can rewrite the equations used in the solution of the model as follows:

\[
y_t + \tau_t - E_t (y_{t+1} + \tau_{t+1}) + (i_t - E_t (\pi_{H,t+1})) = 0 \tag{II.79}
\]

\[
\pi_{H,t} - \lambda \pi_{H,t+1} - \lambda_i (y_t - z_t + \tau_t) = 0 \tag{II.80}
\]

\[
s_t - i_t^* + (i_t - E_t (\pi_{H,t+1})) - E_t (s_{t+1} - \tau_{t+1} + \tau_t) = 0 \tag{II.81}
\]

\[
i_t - \rho_i i_{t-1} - (1 - \rho_i) \chi \pi (E_t (\pi_{H,t+1})) - (1 - \rho_i) \chi \omega (E_t (y_t - z_t + \tau_t)) \tag{II.82}
\]

\[
-\gamma (1 - \rho_i) \chi \pi (E_t (s_{t+1} - s_t)) = 0 \tag{II.83}
\]

\[
\pi_{H,t} - p_{H,t} + p_{H,t-1} = 0 \tag{II.84}
\]

\[
i_t^* = \rho_t i_{t-1} + \varepsilon_t^* \tag{II.85}
\]

\[
\tau_t = \rho_t \tau_{t-1} + \varepsilon_t^\tau \tag{II.86}
\]

\[
z_t = \rho_z z_{t-1} + \varepsilon_t^z \tag{II.86}
\]

where we have set all the constants equal to zero.\(^{23}\) Following the lead of Uhlig (1997), the vector of endogenous state variables is \( x_t = \begin{bmatrix} i_t & p_{H,t} & y_t & s_t \end{bmatrix}' \), the single vector of non-predetermined variable (jump variable) is \( y_t = \begin{bmatrix} \pi_{H,t} \end{bmatrix} \) and the vector of shock variables

\(^{23}\text{Setting all constants equal to zero doesn’t affect our results at all}\)
is $z_t = \begin{bmatrix} i_t^* & \tau_t & z_t \end{bmatrix}$. The model in matrix form is thus:

$$Ax_t + Bx_{t-1} + Cy_t + Dz_t = 0$$

$$E_t \left[ Fx_{t+1} + Gx_t + Hx_{t-1} + Jy_{t+1} + Ky_t + Lz_{t+1} + Mz_t \right] = 0 \quad \text{(II.87)}$$

$$z_{t+1} = Nz_t + \varepsilon_{t+1}$$

In our case, we will rewrite Equation (II.83) in matrix form as follows:

$$Ax_t + Bx_{t-1} + Cy_t + Dz_t = 0 \quad \text{(II.88)}$$

where $A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & -1 & 0 & 0 \end{bmatrix}$, $C = [1]$, and $D = \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}$.

We can write Equations, (II.79), (II.80), (II.81) and (II.82) in matrix form as follows:

$$E_t \left[ Fx_{t+1} + Gx_t + Hx_{t-1} + Jy_{t+1} + Ky_t + Lz_{t+1} + Mz_t \right] = 0 \quad \text{(II.89)}$$

where

$$F = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -\gamma (1 - \rho_i) \chi_\pi \end{bmatrix}, \quad G = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -\lambda_m & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & -(1 - \rho_i) \chi_x & \gamma (1 - \rho_i) \chi_\pi \end{bmatrix}$$

$$H = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -\rho_i & 0 & 0 & 0 \end{bmatrix}, \quad J = \begin{bmatrix} -1 \\ -\lambda_\pi \\ -1 \\ -(1 - \rho_i) \chi_\pi \end{bmatrix}, \quad K = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$
Finally, we can rewrite Equations (II.84), (II.85) and (II.86) in matrix form as follows:

\[
\begin{bmatrix}
0 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
L =
\begin{bmatrix}
0 & 1 & 0 & 0 \\
0 & -\lambda_m & \lambda_m & 0 \\
0 & -1 & -1 & 0 \\
0 & (1 - \rho_i) \chi_x & (1 - \rho_i) \chi_x & 0
\end{bmatrix}.
\]

Finally, we can rewrite Equations (II.84), (II.85) and (II.86) in matrix form as

\[
z_{t+1} = Nz_t + \varepsilon_{t+1}
\]

where

\[
N = \begin{bmatrix}
\rho_{t^*} & 0 & 0 \\
0 & \rho_r & 0 \\
0 & 0 & \rho_z
\end{bmatrix}, \quad \varepsilon_{t+1} = \begin{bmatrix}
\varepsilon_{t+1}^x \\
\varepsilon_{t+1}^r \\
\varepsilon_{t+1}^z
\end{bmatrix}
\]

I. Data Appendix

The data cover the monthly sample over the period 1996:1 to 2006:12. The data sources are the web page of the Bank of Canada (http://www.bankofcanada.ca), the online version of the International Financial Statistics (IFS), and the Energy Information Administration. The details are below.

1. For the data downloaded from the web page of the Bank of Canada:

   (a) Growth rate in total CPI has been used for Canadian inflation.

   (b) Overnight rate has been used for Canadian short-term interest rate.

   (c) Canadian-dollar effective exchange rate index (CERI) has been used for Canadian effective terms of trade.
(d) M1+ (gross) has been used for Canadian M1.

(e) The inflation target has been set to the midpoint of the target range, which is equal to 2.

2. For the data downloaded from online IFS:

(a) Industrial production series (IPS) has been used for Canadian output.

(b) The output gap has been found by detrending Canadian IPS by using Hodrick–Prescott (HP) filter. We use the definition of Khalaf and Kichian (2004) for the measure of output gap. That is, rather than detrending the log of IPS using the full sample, $T$, we proceed iteratively: to obtain the value of the gap at time $t$, we detrend IPS with the data ending in $T$. We then extend the sample by one more observation and re-estimate the trend. This is used to detrend IPS and yields a value for the gap at time $t+1$. This process is repeated until the end of the sample.

(c) For foreign interest rate, government bond yield of the U.S. for 10 years has been used.

3. For the data downloaded from Energy Information Administration:

(a) To get a measure of trade costs, although it is necessary to measure the wedge between the price of imported goods on the domestic market and their price at the source measured in domestic currency units, as a proxy, we use "All Countries Spot Petroleum Price FOB Weighted by Estimated Export Volume (International Dollars per Barrel)" for trade costs. This is the best available data for trade costs to our knowledge. We have also considered using the “Couriers and
Messengers Services Price Index” downloaded from Statistics Canada as an alternative for trade costs. However, the data cover only the period from 2003 to 2006, which is much shorter than our sample period. Nevertheless, from 2003 to 2006, the correlation coefficient between “All Countries Spot Petroleum Price FOB Weighted by Estimated Export Volume” and “Couriers and Messengers Services Price Index” is around 0.95, which can be seen as an indicator of robustness of our analysis.
CHAPTER III

UNDERSTANDING INTERSTATE TRADE PATTERNS

Introduction

What is the main motivation behind intranational trade? Compared to relatively complex models in the literature, this chapter contributes by introducing a simple partial equilibrium model to analyze the motivations behind bilateral trade patterns of regions at the \textit{disaggregate} level. We attempt to find why regions do import more goods from some regions while importing fewer from others. We also investigate why a region imports more of a good while importing less of another one.

In particular, we introduce a type of monopolistic competition model consisting of a finite number of regions. There are two types of goods, namely traded and non-traded. Each region produces and consumes a unique non-traded good. Each region may also consume all varieties of all traded goods, while it can produce only one variety of each traded good. While the traded goods are produced by a perfectly mobile unique factor, the only non-traded good in each region is produced by the same mobile factor together with traded intermediate inputs.

According to this setup, as is standard in the literature, we show that the trade of a variety of a particular traded good across any two regions depend on the relative price of the variety and the total demand (final consumption demand plus intermediate input demand) of the good in the destination (importer) region. Our contribution comes into the picture when we take the ratio between imports of varieties from different sources (exporters). We
find that a region imports more goods (measured in values) from the lower price regions and fewer goods from the more distant regions.

We show that our model together with our estimation methodology has several empirical and analytical benefits compared to gravity models in the following senses:

(i) In our model, there is no identification problem in terms of estimating the elasticity of substitution and the elasticity of distance at the same time. By distinguishing between aggregate level and disaggregate level trade data, together with considering the production side of our model, we can also estimate the elasticity of substitution across goods.

(ii) Our methodology controls for a possible issue of overstating the distance measures (due to using calculated distances, such as great circle distances) mentioned by Hillberry and Hummels (2001).

(iii) By construction, the model is capable of controlling for the effects of local (i.e., wholesale and retail) distribution costs, insurance costs, local taxes, markup differences in production, international trade (under reasonable assumptions), and intermediate input trade, each of which are possible topics for separate debates in the literature (see Anderson and van Wincoop 2004).

(iv) There is an exogenous solution for the estimated trade expression, and thus, there is no need for any income data for estimation, given the technological levels.

We estimate the model using bilateral trade data belonging to the states of the U.S. The estimated parameters correspond to: a) elasticity of substitution across varieties of a good, each produced in a different region; b) elasticity of substitution across goods, each consisting of different varieties; c) elasticity of distance, which governs good specific trade costs; and d) heterogeneity of individual tastes, governing geographic barriers and the
so-called home-bias. We pursue several strategies to estimate these parameters and support our results with different sensitivity analyses. Overall, our model is capable of explaining the interstate trade data up to 84% at the disaggregate level, and up to 77% at the aggregate level.

Our estimated parameters give insights about a number of issues related to interstate trade patterns within the U.S.: a) compared to empirical international studies, elasticity of substitution is lower intranationally; b) compared to empirical international studies, elasticity of distance is higher intranationally; c) there is evidence of home-bias even at the intranational level; d) trade costs are mostly good specific even at the intranational level; e) source-specific fixed effects are important for bilateral trade patterns, effects usually ignored in the literature; f) production technologies are both good and region specific rather than country specific; g) elasticity of substitution across varieties is good specific.

Now, we briefly describe how this study relates to its closest antecedents, especially gravity type studies. The gravity models are popular mostly due to their empirical success.\(^1\) When we look at the theoretical background of gravity type studies, Anderson (1979) is the first one to model gravity equations. The main motivation behind Anderson’s (1979) gravity model is the assumption that each region is specialized in the production of only one good.\(^2\) Despite its empirical success, as Anderson and van Wincoop (2003) point out, the specialization assumption suppresses finer classifications of goods, and thus makes the model useless in explaining the trade data at the disaggregate level. Another deficiency of Anderson’s (1979) gravity model is the lack of a production side. Bergstrand (1985) bridges this gap by introducing a one-factor, one-industry, \(N\)-country general equilibrium model in

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2 In the Appendix of his paper, Anderson (1979) extends his basic model to a model in which multiple goods are produced in each region.
which the production side is considered. In his following study, Bergstrand (1989) extends his earlier gravity model to a two-factor, two-industry, $N$-country gravity model.\textsuperscript{3}

The main deficiency of the gravity models is that they cannot control for good specific transportation costs, good specific local (i.e., wholesale and retail) distribution costs, good specific insurance costs, good specific local taxes, region specific markup differences in production, good specific intermediate input trade or international trade. Moreover, as we have mentioned above, one cannot estimate the elasticity of substitution and the elasticity of distance at the same time by using gravity equations. However, this chapter controls for all of these situations.

None of the papers mentioned above empirically deal with the trade patterns within a country. Recently, Wolf (2000), Hillberry and Hummels (2001), and Millimet and Thomas (2007) bridged this gap by analyzing the interstate trade patterns within the U.S. However, these studies use the aggregate level (i.e., total bilateral) trade data, while this chapter uses disaggregate level bilateral data that give more insight related to good specific analyses. Since they use gravity type models, they also suffer from the same issues mentioned above. Moreover, they cannot distinguish between the elasticity of substitution across varieties of a good, elasticity of substitution across goods, and the elasticity of distance at the same. By taking the ratio between imports of varieties from different origins (exporters), by taking the ratio between imports of different goods, and by including intermediate input trade into the model, this chapter takes care of all of these issues by construction.

Nevertheless, this study is not the first one that considers trade ratios. For instance, studies such as Head and Ries (2001), Head and Mayer (2002), Eaton and Kortum (2004), and Suga (2007) for a monopolistic-competition model of international trade with external economies of scale, Lopez et al. (2006) for an analysis on home-bias on U.S. imports of processed food products, and Gallaway et al. (2003) for an empirical study to estimate short-run and long-run industry-level U.S. Armington elasticities.

\textsuperscript{3} Also see Suga (2007) for a monopolistic-competition model of international trade with external economies of scale, Lopez et al. (2006) for an analysis on home-bias on U.S. imports of processed food products, and Gallaway et al. (2003) for an empirical study to estimate short-run and long-run industry-level U.S. Armington elasticities.
(2002), and Romalis (2007), among others, have somehow also considered trade ratios in their gravity type models. Most of these studies have attempted to eliminate price measures from the gravity equation since they see them as nuisances. In order to get rid of those price measures, one cannot simply take the ratio among imports of varieties from different origins; they also have to consider the ratio among imports of varieties within regions. This process results with having an index for freeness of trade that helps us determine the impacts of borders (mostly related to international trade literature) rather than explaining the intranational trade patterns. Although this approach seems fine up to a point, it has the deficiency of not considering the production side at all and not having a structure to analyze the disaggregate level trade. The closest study to this study is by Romalis (2007). However, by eliminating the source specific marginal costs (i.e., the production side), Romalis (2007) cannot identify the elasticity of substitution and the elasticity of distance at the same time; instead, he can only estimate the elasticity of substitution. By considering the production side, this chapter can estimate the elasticity of substitution and the elasticity of distance at the same time. Moreover, all of these studies also don’t take into account zero trade observations that have a high share in overall observations.\textsuperscript{4} This chapter contributes to the literature by controlling for all of these issues, therefore by having more accurate empirical results.

The rest of the chapter is organized as follows. Section 2 introduces our regional trade model. Section 3 provides insights and depicts our estimation methodology. Section 4 gives the empirical results, while Section 5 concludes. The data are described in the\textsuperscript{4}Helpman et al. (2008) show that almost 50\% of the observations are zero trade observations in international trade.
Appendix.

The Model

We model an economy consisting of a finite number of regions. In each region, there are two types of goods, namely traded and non-traded. While a unique non-traded good is produced and consumed within all regions (thus, the non-traded goods market is in equilibrium in each region separately), each region may consume all varieties of all traded goods and can produce only one variety of each traded good. Since we only care about the partial equilibrium bilateral trade implications of our model, in many instances, we skip the irrelevant details of the model in order to keep it as simple as possible.\footnote{A general equilibrium framework is not necessary in our analysis. It would only complicate our model with unnecessary details.}

Each traded good is denoted by $j = 1, \ldots, J$. Each variety is denoted by $i$ that is also the notation for the region producing that variety. We make our analysis for a typical region, $r$. In the model, generally speaking, $H_{a,b}(j)$ stands for the variable $H$, where $a$ is related to the region of consumption, $b$ is related to the variety (and thus, the region of production), and $j$ is related to the good.

Individuals

The representative agent in region $r$ maximizes utility $U(C_T^r, C_{NT}^r)$ where $C_T^r$ is a composite index of traded goods and $C_{NT}^r$ is a unique non-traded good. The composite index of traded goods, $C_T^r$, is given by:

$$C_T^r \equiv \left( \sum_j (\gamma_j)^{\frac{1}{\kappa}} (C^T_r(j))^{\frac{\kappa-1}{\kappa}} \right)^{\frac{\kappa}{\kappa-1}}$$
where $C_T^r(j)$ is given by:

$$C_T^r(j) \equiv \left( \sum_i (\beta_r^i \theta_i)^{\frac{1}{\eta(j)}} \right)^{\frac{1}{\eta(j)-1}} \left( C_{r,i}^T(j) \right)^{\frac{\eta(j)-1}{\eta(j)-1}}$$

where $C_{r,i}^T(j)$ is the variety $i$ of traded good $j$ imported from region $i$; $\varepsilon > 0$ is the elasticity of substitution across goods; $\eta(j) > 1$ is the elasticity of substitution across varieties of good $j$; $\gamma_j$ is a good specific taste parameter; $\beta_r$ is a destination (i.e., importer) specific taste parameter; and finally, $\theta_i$ is a source (i.e., exporter) specific taste parameter. For different varieties, while having only one bilateral taste parameter, which is both destination and source specific, is standard in the literature, decomposing it into $\beta_r$ and $\theta_i$ is new in this study.\(^6\) In particular, both $\beta_r$ and $\theta_i$ can be used as fixed effects in a regression analysis; i.e., they together represent a unique bilateral taste parameter between regions $r$ and $i$. Moreover, by putting restrictions on $\theta_i$, one can easily measure home-bias implications of our model. Besides, one can also control for issues such as migration by using $\theta_i$ (see Millimet and Thomas, 2007). Our claim will be clearer when we show the bilateral trade implications of our model in Section 3. We will also test for the validity of this assumption in Section 4.

The optimal allocation of any given expenditure within each variety of goods yields the following demand functions:

$$C_{r,i}^T(j) = \beta_r^i \theta_i \left( \frac{P_{r,i}^T(j)}{P_r^T(j)} \right)^{-\eta(j)} C_r^T(j) \quad \text{(III.1)}$$

and

$$C_r^T(j) = \gamma_j \left( \frac{P_r^T(j)}{P_r^T(j)} \right)^{-\varepsilon} C_r^T \quad \text{(III.2)}$$

where $P_r^T(j) \equiv \left( \sum_i \beta_r^i \theta_i P_{r,i}^T(j) \right)^{1-\eta(j)}$ is the price index of the traded good $j$ (which

\(^6\)Distinguishing between destination and source specific taste parameters has useful properties in terms of our estimation. Our reason will be clearer when we move to Section 3.
is composed of different varieties), and $P^T_r \equiv \left( \sum_i \gamma_j P^T_r (j)^{1-\varepsilon} \right)^{1/\varepsilon}$ is the cost of living index in region $r$. It is implied that $P^T_r (j) C^T_r (j) = \sum_j P^T_{r,i} (j) C^T_{r,i} (j)$.

**Firms**

Since there are two types of goods, namely traded and non-traded, there are two types of firms in each region.

**Production of Traded Goods**

Traded good $j$ in region $r$ (i.e., variety $r$ of good $j$) is produced by the following production function:

$$Y^T_r (j) = A_r (j) L^T_r (j) \quad \text{(III.3)}$$

where $A_r (j)$ represents the good and region specific technology, and $L^T_r (j)$ represents a completely mobile factor of production of which hour is worth $W$ in all regions.\(^7\)

The firm chooses $L^T_r (j)$, taking as given its price $W$. The cost minimization problem of the firm implies that the marginal cost of producing variety $r$ of good $j$ (in region $r$) is given by:

$$MC^T_r (j) = \frac{W}{A_r (j)} \quad \text{(III.4)}$$

Note that $MC^T_r (j)$ is good and region specific.

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\(^7\)One can easily assume $L_r (j)$ to be labor and/or capital, but our results are not affected at all by these details, because we don’t employ any factor market in order to keep the model as simple as possible in our analysis.
Production of Non-traded Goods

The unique non-traded good in region \( r \) is produced by a production function \( Y_r^{NT} \left( L_r^{NT}, G_r^{NT} \right) \) where \( L_r^{NT} \) represents the completely mobile factor of production (i.e., the same factor used in the production of traded goods) and \( G_r^{NT} \) is the counterpart of \( C_r^T \) given by:

\[
G_r^{NT} \equiv \left( \sum_j (\gamma_j) \left( G_r^{NT} (j) \right)^{\frac{\varepsilon - 1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon - 1}}
\]

where \( G_r^{NT} (j) \) is given by:

\[
G_r^{NT} (j) \equiv \left( \sum_i (\beta_r \theta_i) \left( G_r, \theta_i (j) \right)^{\frac{\eta(j) - 1}{\eta(j)}} \right)^{\frac{\eta(j)}{\eta(j) - 1}}
\]

The optimal allocation of any given expenditure within each variety of intermediate inputs yields the following demand functions:

\[
G_{r,i}^{NT} (j) = \beta_r \theta_i \left( \frac{P_r^{T,i} (j)}{P_r^{T,i} (j)} \right)^{-\eta(j)} G_r^{NT} (j)
\]

(III.5)

and

\[
G_r^{NT} (j) = \gamma_j \left( \frac{P_r^{T} (j)}{P_r^{T}} \right)^{-\varepsilon} G_r^{NT}
\]

(III.6)

Note that the firms share the same taste parameters, \( \beta_r, \theta_i \) and \( \gamma_j \), with the individuals. Although this is somehow a restrictive assumption, it has very nice properties in terms of bilateral trade implications that are discussed in Section 3.

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8Since we care about the bilateral trade implications of our model, after assuming that the nontraded goods market is in equilibrium in each region, the exact functional forms of \( Y_r^{NT} \left( L_r^{NT}, G_r^{NT} \right) \), demand for \( L_r^{NT} \), and the marginal cost implication for the nontraded goods are all irrelevant in our analysis.
Trade Cost

We have to define our trade cost first. Anderson and van Wincoop (2004) categorize the trade costs under two names, costs imposed by policy (tariffs, quotas, etc.) and costs imposed by the environment (transportation, wholesale and retail distribution, insurance against various hazards, etc.). Since we analyze trade within a country (i.e., the U.S.), we ignore the first category and focus on the second one. Instead of assuming an iceberg transport cost, we assume that the transportation is achieved by a transportation sector, which is not modeled here. This assumption is important to distinguish between the export income received by the exporter and the transportation income received by the transporter. The implications of this assumption will be clearer below. In particular, we assume that, if there is a trade between regions, it is subject to a transportation cost:

\[
P_{i,r}^{T} (j) = (1 + \tau_{i,r} (j)) \left( P_{r,r}^{T} (j) \right)
\]

(III.7)

where \( P_{r,r}^{T} (j) \) is the price of the traded good at the factory gate (i.e., the source); \( \tau_{i,r} (j) > 0 \) is a good specific net transportation cost from region \( r \) to region \( i \); \( D_{i,r} \) is the distance between regions \( r \) and \( i \); and \( \delta (j) \) is the elasticity of distance. This assumption is commonly used in the literature (see Anderson and van Wincoop 2003, 2004). The cost implications of our model in terms of wholesale distribution, retail distribution, insurance or local taxes, will be provided below.

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9Since we consider only the partial equilibrium bilateral trade implications of our model, the actual role/model of the transportation sector is irrelevant in our analysis after assuming that transportation is achieved by using the completely mobile factor of production (i.e., the same factor used in the production of traded/nontraded goods).

10Needless to say, the existence of trade is determined by Equation III.1 for all \( i, r \) and \( j \). As we will discuss in detail in the following sections, we consider the absence of trade, besides the existence of it in our empirical analysis.

11For the distance within each state (i.e., the internal distance), we use the proxy developed by Wei (1996), which is one-fourth the distance of a region’s capital from the nearest capital of another region.
Equilibrium

Since we care about the partial equilibrium bilateral trade implications of our model, we naturally assume that the non-traded good market, of which details are not shown here, is in equilibrium in each region. So, in this subsection, we depict equilibrium in the traded goods market. In particular, for each variety $r$ of traded good $j$ produced in region $r$, the market clearing condition implies:

$$Y^T_T(r)(j) = \sum_i C^T_{i,r}(j) + \sum_i G^{NT}_{i,r}(j) \quad \text{(III.8)}$$

where $C^T_{i,r}(j)$ is the demand of region $i$ for variety $r$ of traded good $j$ (produced in region $r$); and $G^{NT}_{i,r}(j)$ is the intermediate input demand for variety $r$ of traded good $j$ (produced in region $r$) demanded for the production of the non-traded good in region $i$. Equation III.8 basically says that variety $r$ of final good $j$ produced in region $r$ is either consumed locally or by other regions, either for final consumption or as an intermediate input.

Price Setting

Since we care about the partial equilibrium bilateral trade implications of our model, the price setting behavior of the firms producing the unique non-traded good in each region is irrelevant in our analysis. For the traded goods, in region $r$, we assume that a typical firm that produces variety $r$ of traded good $j$ faces the following profit maximization problem:

$$\max_{P^T_{r,j}} Y^T_T(r)(j) \left[ P^T_{r,j}(j) - MC^T_r(j) \right]$$
subject to Equation III.8 and the symmetric versions of Equation III.1 and III.5. The first order condition for this problem is as follows:\(^\text{12}\)

\[
Y_r^T (j) \left[ 1 - \frac{\eta(j)}{P^T_{r,r} (j)} \left( P^T_{r,r} (j) - MC^T_r (j) \right) \right] = 0
\]

which implies that:

\[
P^T_{r,r} (j) = \left( \frac{\eta(j)}{\eta(j) - 1} \right) MC^T_r (j) = \left( \frac{\eta(j)}{\eta(j) - 1} \right) W \frac{A_r(j)}{A_r(j)}
\]

where \(\frac{\eta(j)}{\eta(j) - 1}\) represents the gross mark-up. For the second line, we have used Equation III.4 which implies that, for a specific good, the factory price of the product differs in each region only because of the region specific technology levels.

**Bilateral Trade**

We distinguish between disaggregate and aggregate level trade in our analysis.

While the disaggregate level trade considers bilateral ratios of imports of a region for different varieties of a particular good, the aggregate level trade considers bilateral ratios of imports of different goods for a particular region.

**Disaggregate Level Trade**

By using Equations III.1, III.5 and III.7, we obtain our key expression for the ratio of imports of region \(r\) across regions \(a\) and \(b\), which is expressed by:

\[
\frac{X_{r,a} (j)}{X_{r,b} (j)} = \frac{\theta_a}{\theta_b} \left( \frac{P^T_{b,b} (j)}{P^T_{a,a} (j)} \right)^{\eta(j) - 1} \left( \frac{D_{r,b}}{D_{r,a}} \right)^{\delta(j)\eta(j)}
\]

\(^\text{12}\)Notice that the firm takes the composite consumption index of good \(j\) (i.e., \(C^T_r (j)\)), the composite index of intermediate demand for good \(j\) (i.e., \(G^N^T_r (j)\)) and the composite price index of good \(j\) (i.e., \(P^T_r (j)\)) in each region as given in the optimization problem.
where \( X_{r,k}(j) = \left( C_{r,k}^T(j) + G_{r,k}^{NT}(j) \right) P_{k,k}^T(j) \) is the value of total imports of region \( r \) from region \( k \) measured at the source for good \( j \). Equation III.10 says that a region imports more goods (measured in values) from the lower price regions and fewer goods from the more distant regions.

Substituting Equation III.9 into Equation III.10 results in the general form of our estimated equations for our disaggregated level trade analysis:

\[
\frac{X_{r,a}(j)}{X_{r,b}(j)} = \frac{\theta_a}{\theta_b} \left( \frac{A_a(j)}{A_b(j)} \right)^{\eta(j)-1} \left( \frac{D_{r,b}}{D_{r,a}} \right)^{\delta(j)\eta(j)}
\]

(III.11)

Note that Equation III.11 is an exogenous solution for our estimated disaggregate level trade expression, and thus, there is no need for any endogenous data such as income for the estimation given the technology levels. Moreover, it can easily be estimated in log terms.

**Aggregate Level Trade**

By using Equations III.2 and III.6, we obtain our key expression for the ratio of imports of region \( r \) in terms of goods \( j \) and \( k \) as follows:

\[
\frac{X_r(j)}{X_r(k)} = \frac{\gamma_j}{\gamma_k} \left( \frac{P_r^T(j)}{P_r^T(k)} \right)^{1-\varepsilon}
\]

(III.12)

where \( X_r(m) = (C_r^T(m) + G_r^{NT}(m)) P_r^T(m) = \sum_i \left( (C_{r,i}^T(m) + G_{r,i}^{NT}(m)) P_{r,i}^T(j) D_{r,i}^{\delta(j)} \right) \) is the value of total imports of region \( r \) in terms of good \( m \) measured at the destination. Equation III.12 says that a region imports more (less) of a good which has a lower (higher) destination price.

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13If we had an iceberg cost in our analysis, we would have had \( X_{r,k}(j) = (C_{r,k}^T(j) + G_{r,k}^{NT}(j)) P_{k,k}^T(j)(1 + \tau_{r,k}(j)) \) as the export income received by the exporter region for good \( j \). However, this is not the case in the real world that distinguishes between the exporter sector and the transportation sector. For instance, our data set of Commodity Flow Survey includes only the export income received by the firms, not the transportation income.

14Since the data set of Commodity Flow Survey includes only the export income received by the firms, we have to distinguish between the value of exports at the source and at the destination. We will control for these issues in our empirical analysis below.
Substituting Equations III.7, III.9, and $P_T^r(j) \equiv \left( \sum_i \beta_i \theta_i P_T^{r;i}(j)^{1-\eta(j)} \right)^{\frac{1}{1-\eta(j)}}$ into Equation III.12 results in the general form of our estimated equations for our aggregate level trade analysis:

$$\frac{X_r(j)}{X_r(k)} = \gamma_j \left( \frac{\frac{\eta(j)}{\eta(j)-1}}{\frac{\eta(k)}{\eta(k)-1}} \left( \sum_i \beta_i \theta_i D^{r;i}_{r;i} \left( A_i(j) \right)^{\eta(j)-1} \right)^{\frac{1}{1-\eta(j)}} \right)^{1-\varepsilon}$$

(III.13)

Note that Equation III.13 is an exogenous solution for our estimated aggregate level trade expression, and thus, there is again no need for any endogenous data such as income for the estimation given the technology levels. Moreover, it can easily be estimated in log terms after estimating the disaggregate level expression given by Equation III.11 (i.e., after obtaining estimates for $\eta(j)$’s and $\delta(j)$’s).

**Remarks and Estimation Methodology**

We employ a two-step estimation process. First, we test the empirical power of our model at the disaggregate level and obtain estimates of elasticity of substitution across varieties of each good (i.e., $\eta(j)$’s), and good specific distance elasticities (i.e., $\delta(j)$’s) in the disaggregate level estimation, by which we obtain good specific price indices (i.e., $P_i(j)$’s). Second, we test the empirical power of our model at the aggregate level and obtain the elasticity of substitution across goods (i.e., $\varepsilon$).

**Disaggregate Level Trade Estimation**

In this subsection, we provide the implications of Equation III.11, and we empirically test different log versions of it by using the Commodity Flow Survey (CFS) that covers bilateral interstate trade data within the U.S. The details of data are described in
Although Equation III.11 holds on average, it doesn’t hold for each bilateral trade ratio. In empirical terms, following Santos Silva and Tenreyro (2006), and Henderson and Millimet (2008), to address the unobservable nature of bilateral trade ratios, we assume that there is an error term associated with each ratio, which implies that:

\[
\frac{X_{r,a}(j)}{X_{r,b}(j)} = \frac{\theta_a}{\theta_b} \left( \frac{A_{a}(j)}{A_{b}(j)} \right)^{\eta(j)-1} \left( \frac{D_{r,b}}{D_{r,a}} \right)^{\delta(j)\eta(j)} + \mu_{r,a,b,j}
\]

where \( E \left[ \mu_{r,a,b,j} \left| \frac{\theta_a}{\theta_b}, \frac{A_{a}(j)}{A_{b}(j)}, \frac{D_{r,b}}{D_{r,a}} \right. \right] = 0 \). This can be rewritten as:

\[
\frac{X_{r,a}(j)}{X_{r,b}(j)} = \frac{\theta_a}{\theta_b} \left( \frac{A_{a}(j)}{A_{b}(j)} \right)^{\eta(j)-1} \left( \frac{D_{r,b}}{D_{r,a}} \right)^{\delta(j)\eta(j)} v_{r,a,b,j}
\]

where

\[
v_{r,a,b,j} = 1 + \frac{\mu_{r,a,b,j}}{\frac{\theta_a}{\theta_b} \left( \frac{A_{a}(j)}{A_{b}(j)} \right)^{\eta(j)-1} \left( \frac{D_{r,b}}{D_{r,a}} \right)^{\delta(j)\eta(j)}}
\]

and \( E \left[ v_{r,a,b,j} \left| \frac{\theta_a}{\theta_b}, \frac{A_{a}(j)}{A_{b}(j)}, \frac{D_{r,b}}{D_{r,a}} \right. \right] = 1 \). Taking the log of both sides in Equation III.14 results in the following log-linear expression for the bilateral disaggregate level trade ratios:

\[
\log \left( \frac{X_{r,a}(j)}{X_{r,b}(j)} \right) = \log \left( \frac{\theta_a}{\theta_b} \right) + (\eta(j) - 1) \log \left( \frac{A_{a}(j)}{A_{b}(j)} \right) + \delta(j) \eta(j) \log \left( \frac{D_{r,b}}{D_{r,a}} \right) + \log \left( v_{r,a,b,j} \right)
\]

To obtain a consistent estimator of the slope parameters by the Ordinary Least Squares (OLS), we assume that \( E \left[ \log \left( v_{r,a,b,j} \right) \left| \frac{\theta_a}{\theta_b}, \frac{A_{a}(j)}{A_{b}(j)}, \frac{D_{r,b}}{D_{r,a}} \right. \right] \) does not depend on the regressors.\(^{15}\)

\(^{15}\)It is well known that modeling zero interregional flows using a normal error process leads to problems. If the dependent variable cannot take a value below zero, then a normal error process is a poor approximation. Nevertheless, we don’t have such a concern, because our log-linearized equation does have values below zero, by considering the (log) ratio of bilateral trade values.
Because of Equation III.15, this condition is met only if \( \mu_{r,a,b,j} \) can be written as follows:

\[
\mu_{r,a,b,j} = \frac{\theta_a}{\theta_b} \left( \frac{A_a(j)}{A_b(j)} \right)^{\eta(j)-1} \left( \frac{D_{r,b}}{D_{r,a}} \right)^{\delta(j)\eta(j)} \xi_{r,a,b,j}
\]

where \( \xi_{r,a,b,j} \) is a random variable statistically independent of the regressors. In such a case, \( v_{r,a,b,j} = 1 + \xi_{r,a,b,j} \) and therefore is statistically independent of the regressors, implying that \( E \left[ \log (v_{r,a,b,j}) \middle| \frac{\theta_a}{\theta_b}, \frac{A_a(j)}{A_b(j)}, \frac{D_{r,b}}{D_{r,a}} \right] \) is a constant. Following Santos Silva and Tenreyro (2006), we relax the assumption of \( E \left[ \log (v_{r,a,b,j}) \middle| \frac{\theta_a}{\theta_b}, \frac{A_a(j)}{A_b(j)}, \frac{D_{r,b}}{D_{r,a}} \right] \) not depending on the regressors, in our Sensitivity Analysis #4, below, by considering the Poisson Pseudo-Maximum Likelihood (PPML) estimator.

In our estimation, we run only one OLS (or PPML) regression for the pooled sample by including relevant dummy variables for \( \frac{\theta_a}{\theta_b} \), \( \eta(j) \) and \( \delta(j) \) in Equation III.16. Although \( A_i(j) \)'s are region and good specific technology levels in Equation III.16, they don’t necessarily capture all the source specific fixed effects. This is why source specific taste parameters (i.e., \( \theta_i \)'s) may play an important role in our estimation. For instance, in additional to the technology levels, source specific fixed effects may capture possible differences in source specific production markups, source specific production taxes, and so on. We test the validity of having both these fixed effects and technology levels at the same time in Version B and Version G of our empirical estimation, below.\(^{16}\)

According to Equation III.16, the following propositions are implied:

**Proposition 1** Both \( \delta(j) \) and \( \eta(j) \) can be identified in Equation III.16 which is not the

\(^{16}\)Multicollinearity is less of a problem in a cross-sectional analysis like ours that has a high sample size. The reasoning is that we run only one regression instead of good specific regressions; if we were running good specific regressions, then \( \theta_i \)'s and \( A_i(j) \)'s would have been perfectly correlated, because, in such a case, we would have good specific \( \theta_i \)'s. Moreover, the individual effects of technology and source specific taste parameters can both be assessed when there are sufficient number of observations of high technology regions with low fixed effects and low technology regions with high fixed effects. Besides, the theoretical consequences of multicollinearity is still a debate, because even if the multicollinearity is very high, the OLS estimators still remain to be the best linear unbiased estimators. The only possible problem arises due to having wide confidence intervals in the presence of multicollinearity. However, by having very low confidence intervals, our estimation results below are robust to a possible multicollinearity problem. See Achen (1982) and Gujarati (1995) for more details.
case in most gravity models (see Anderson and van Wincoop 2003; Hummels 1999, 2001; Wei 1996).

**Proof.** The identification is realized via the technology levels which are usually ignored in gravity models. In particular, since both \((\eta(j) - 1)\) and \(\delta(j) \eta(j)\) can be estimated by Equation III.16, one can identify both \(\eta(j)\) and \(\delta(j)\) while also calculating their standard errors by employing the Delta method. ■

**Proposition 2** All the variables in Equation III.16 are exogenous, which leaves an applied researcher free from a possible endogeneity problem. Moreover, there is no need for income data given the exogenous technology levels.

**Proof.** The proof follows through Equation III.11.\(^{17}\)

**Proposition 3** Assuming that overstatement of a distance is proportional to the distance itself, the model controls for such an issue (because of the use of calculated distance measures such as great circle distances) as mentioned by Hillberry and Hummels (2001).

**Proof.** Assuming that overstatement of a distance is proportional to the distance itself, the distance ratio in Equation III.16 is not affected at all. See sensitivity analysis #3 in Section 4 for details. ■

**Proposition 4** By construction, the model is capable of controlling for the effects of local (i.e., wholesale and retail) distribution costs, insurance costs or local taxes, each of which are possible topics for separate debates in the literature (see Anderson and van Wincoop 2004).

**Proof.** To see this, consider Equation III.1 by including such possible good specific proportional costs. For instance, say that there is a proportional (net) cost of \(\varphi(j)\) for good \(j\) in region \(r\). Then, it follows that:

\[
C_{r,a}(j) = \theta_a \left( \frac{P_{r,a}(j) (1 + \varphi(j))}{P_r(j) (1 + \varphi(j))} \right)^{-\eta(j)} C_r(j)
\]

\(^{17}\)If trade leads to technology transfer, than technology may be correlated with past trade levels. And, if there are unobservables omitted that are serially correlated, then technology will be endogenous. Nevertheless, these are not issues in our case, because we have a static rather than a dynamic analysis. Moreover, we have unobservable (source specific, destination specific and bilateral specific) fixed effects in our analysis.
and
\[ C_{r,b}(j) = \theta_b \left( \frac{P_{r,b}(j)(1+\varphi(j))}{P_r(j)(1+\varphi(j))} \right)^{-\eta(j)} C_r(j) \]

The same logic applies for Equation III.5, which together with the expressions above, implies exactly the same expression as in Equation III.10. ■

**Proposition 5** By construction, the model is capable of controlling for the effects of intermediate input trade.

**Proof.** Proof follows through definition of \( X_{r,k}(j) = \left( C_{r,k}^T(j) + G_{r,k}^T(j) \right) P_{k,k}^T(j) \) in Equation III.10. ■

Under certain assumptions, the model is also capable of controlling for the effects of international trade. In particular, we assume that the international trade partners of the U.S. share similar tastes with the states in which the customs are located. Our justification comes from the fact that, in CFS, international export (import) shipments are included, with the domestic destination (source) defined as the U.S. port, airport, or border crossing of exit from the U.S. After this reasonable assumption, it follows that our estimated trade ratio given by Equation III.16 is not affected at all by international trade, since the inclusion of international trade will be proportional in such a case.

By using the general form in Equation III.16, we test several restricted versions of it along with its unrestricted version. These restrictions are not only important for econometric significance tests, but they are also important for economic intuition in terms of the contribution of each variable in Equation III.16 to explain the interstate trade patterns. In particular, we test for the following versions of Equation III.16:

Version A) Unrestricted version of Equation III.16 in which we estimate \( \eta^A \) (the vector consisting of \( \eta(j) \)'s), \( \theta^A \) (the vector consisting of \( \frac{\theta_a}{\theta_b} \)'s), and \( \delta^A \) (the vector consisting
of $\delta (j)$’s for all $r, j, a$ and $b$. This is our benchmark equation by which we use $\frac{\theta_a}{\theta_b}$ values as fixed effects in our regression, by which we can estimate $\eta (j)$’s, by which we can estimate $\delta (j) \eta (j)$’s, and thus, by which we can obtain estimates of $\delta (j)$ and relative standard errors through the use of the Delta method.

Version B) Restricted version of Equation III.16 in which $\theta_i = \theta$ for all $i$, thus, in which we estimate $\eta^B$ (the vector consisting of $\eta (j)$’s), and $\delta^B$ (the vector consisting of $\delta (j)$’s) for all $j$. Recall that in the unrestricted version of Equation III.16, $\theta_i$ values serve as source specific fixed effects in the regression analysis. When $\theta_a = \theta_b$, it follows that $\log \left( \frac{\theta_a}{\theta_b} \right) = 0$. Thus, the purpose of this restricted version is that it helps us evaluate whether or not there are source specific fixed effects. This is also important in terms of testing our assumption of source specific taste parameters in our CES consumption/intermediate input functions. We can also see the contribution of these fixed effects in explaining the interstate trade patterns by comparing the results of this version with the results of version A through an additional restriction test.

Version C) Restricted version of Equation III.16 in which $\delta (j) = \delta$ and $\eta (j) = \eta$ for all $j$, and thus, in which we estimate $\eta, \delta$ and $\theta^C$ (the vector consisting of $\frac{\theta_a}{\theta_b}$’s) for all $r, a$ and $b$. The purpose of this restriction is that it helps us decide whether or not the trade costs and elasticities of substitution across varieties are good specific. This restriction is important, because most of the gravity type studies ignore good specific variations that affect the accuracy of the estimation results. Together with Version H, this restriction is also used to figure out whether or not the trade costs are good specific.

Version D) Restricted version of Equation III.16 in which $\theta_i = \theta$ for all $i$; and in which
\( \delta(j) = \delta \) and \( \eta(j) = \eta \) for all \( j \); thus, in which we estimate \( \eta \) and \( \delta \). This restriction is used to test whether or not there are source specific taste parameters when there are common trade costs and common elasticity of substitution across varieties for different goods.

**Version E**) Restricted version of Equation III.16 in which \( \frac{\theta}{\theta^H} = \theta^H \) and \( \frac{\theta}{\theta^H} = 1 \) for all \( r, a(\neq r), b(\neq r) \); and in which \( \delta(j) = \delta \) and \( \eta(j) = \eta \) for all \( j \), thus, in which we estimate \( \theta^H, \eta \) and \( \delta \). Since we make our analysis for a typical region \( r \), \( \frac{\theta^G}{\theta^H} = \theta^H \) and \( \frac{\theta^G}{\theta^H} = 1 \) together means that the goods purchased within a region are different from the goods imported from other regions, i.e., the so-called *home-bias*. Together with \( \delta(j) = \delta \) and \( \eta(j) = \eta \), the main purpose of this restriction is to find whether or not there is any home-bias, even at the intranational level, when trade costs and elasticities of substitution across varieties are the same across goods.

**Version F**) Restricted version of Equation III.16 in which \( \frac{\theta}{\theta^H} = \theta^H \) and \( \frac{\theta}{\theta^H} = 1 \) for all \( r, a(\neq r), b(\neq r) \); thus, in which we estimate \( \theta^H, \eta^F \) (the vector consisting of \( \eta(j)'s \)) and \( \delta^F \) (the vector consisting of \( \delta(j)'s \) for all \( j \). This is the same as version E except that the trade costs are now good specific. Thus, the main purpose of this restriction is to find whether or not there is any home-bias, even at intranational level, when elasticities of substitution across varieties, and trade costs are good specific.

**Version G**) Restricted version of Equation III.16 in which \( A_a(j) = A_b(j) \) for all \( j \) (which is equivalent, since we talk about the ratios, saying that \( A_i(j) = A \) for all \( i \) and \( j \)); thus, in which we estimate \( \eta^G, \delta^G \) (the vector consisting of \( \eta(j) \delta(j)'s \)) and \( \theta^G \) (the vector consisting of \( \frac{\theta_a}{\theta^H} \)'s). The purpose of this restriction is to evaluate whether the technology levels are region specific or country specific.
Version II) Restricted version of Equation III.16 in which \( \eta(j) = \eta \) for all \( j \), thus, in which we estimate \( \eta, \theta^A \) (the vector consisting of \( \frac{\partial \theta}{\partial \theta} \)'s), and \( \delta^A \) (the vector consisting of \( \delta(j) \)'s) for all \( r, j, a \) and \( b \). The purpose of this restriction is that it helps us decide whether or not the elasticity of substitution across varieties is good specific. This restriction is important, because most of the gravity type studies ignore good specific \( \eta(j) \)'s which affect the accuracy of the estimation results.

**Aggregate Level Trade Estimation**

In this section, we introduce our methodology to estimate Equation III.13. We empirically test it by using CFS data set and the estimation results of the disaggregate level trade estimation. Analogous to the disaggregate level trade equation, although Equation III.13 holds on average, it doesn’t hold for each bilateral trade ratio. Therefore, we assume that there is an error term associated with each ratio, which implies that:

\[
\frac{X_r(j)}{X_r(k)} = \frac{\gamma_j}{\gamma_k} \left( \frac{\eta(j)^{\lambda(j)-1}}{} \left( \sum_i \beta_r \theta_i D_{r,i}^{\delta(j)} (A_i (j))^{\eta(j)-1} \right) \right) ^{\lambda(j)} + \mu_{r,j,k}
\]

where \( \left[ \mu_{r,j,k} \left|^{\gamma_j} \gamma_k \right. \right] \left( \frac{\eta(j)^{\lambda(j)-1}}{} \left( \sum_i \beta_r \theta_i D_{r,i}^{\delta(j)} (A_i (j))^{\eta(j)-1} \right) \right) ^{\lambda(j)} = 0. \) This can be rewritten as:

\[
\frac{X_r(j)}{X_r(k)} = \frac{\gamma_j}{\gamma_k} \left( \frac{\eta(j)^{\lambda(j)-1}}{} \left( \sum_i \beta_r \theta_i D_{r,i}^{\delta(j)} (A_i (j))^{\eta(j)-1} \right) \right) ^{\lambda(j)} + \nu_{r,j,k} \quad (III.17)
\]

where

\[
\nu_{r,j,k} = 1 + \frac{\mu_{r,j,k}}{\left( \frac{\gamma_j}{\gamma_k} \right) \left( \frac{\eta(j)^{\lambda(j)-1}}{} \left( \sum_i \beta_r \theta_i D_{r,i}^{\delta(j)} (A_i (j))^{\eta(j)-1} \right) \right) ^{\lambda(j)}}
\]

(III.18)
and \( E \left[ u_{r,j,k} \right] \) is a constant. As in the assumption of disaggregate level analysis, for robustness, in addition to the OLS regression, we relax the regressors. Because of Equation III.18, this condition is met only if \( \mu_{r,j,k} \) can be written as follows:

\[
\mu_{r,j,k} = \frac{\gamma_j}{\gamma_k} \left( \frac{\eta(j)}{\eta(j)-1} \right) \left( \frac{1}{\eta(j)} \right) \left( \frac{1}{\eta(k)} \right)
\]

where \( \xi_{r,j,k} \) is a random variable statistically independent of the regressors. In such a case, \( u_{r,j,k} = 1 + \xi_{r,j,k} \) and therefore is statistically independent of the regressors, implying that \( E \left[ \log(u_{r,j,k}) \right] \) is a constant. As in the disaggregate level analysis, for robustness, in addition to the OLS regression, we relax the assumption of \( E \left[ \log(u_{r,a,b,j}) \right] \) not depending on the regressors, by considering a PPML regression.

In our estimation, we run only one OLS (or PPML) regression for the pooled sample by including relevant dummy variables for each \( \frac{\gamma_j}{\gamma_k} \) in Equation III.19. After having the estimates for \( \eta(j) \)'s and \( \delta(j) \)'s coming from the disaggregate level estimation, we have data and parameters for everything in Equation III.19 except for \( \beta_i \)'s and \( \theta_i \)'s. In particular,
\( \beta_i \)'s cannot be estimated by our disaggregate level analysis, because they are cancelled out after considering trade ratios. Moreover, we cannot uniquely identify each and every \( \theta_i \) in our disaggregate level analysis due to overidentification issues. Hence, we restrict ourselves to a special case in which \( \beta_i = \theta_i = 1 \) for all \( i \) in our aggregate level analysis.

Although calculated \( P_i(j) \)'s are region and good specific price levels in Equation III.19, they don't necessarily capture all the good specific fixed effects, especially the actual preferences of the individuals for specific goods. This is why good specific taste parameters (i.e., \( \gamma_i \)'s) may play an important role in our estimation. Below, we test the validity of having both these fixed effects and price levels at the same time.

**Empirical Results**

The empirical results for disaggregate and aggregate level trade estimations are given in the following subsections. Before we continue, we have to take care of one more issue: How should we include zero trade observations in our log-linear estimated equation? For the sensitivity of our analysis, we follow three different approaches: 1) Assume that zero (trade) observations are equal to one U.S. dollar’s worth; 2) assume that zero (trade) observations are equal to one U.S. cent’s worth; 3) ignore the zero (trade) observations.\(^{18}\)

Although the last one will be biased toward low elasticities of substitution compared to the other two, it is worth presenting it for the sake of sensitivity. Moreover, we also use that third approach to compare the effects of using great circle distances and actual CFS distances, which is mentioned by Hillberry and Hummels (2001). The estimation based on

\(^{18}\)Unfortunately, we cannot employ a tobit specification to account for the zeros, because we consider the trade ratios instead of the trade itself. In particular, when we have a zero trade observation, then either the numerator or the denominator of the left hand side of Equation III.16 (or both) is equal to zero. This would make the trade ratio equal to either zero or infinity (or indeterminate), and thus, employing a tobit specification would not be plausible in log terms.
the first approach will be presented as the Benchmark Case, and the estimation based on the others will be presented as the Sensitivity Analyses.

**Disaggregate Level Trade Estimation Results**

The disaggregate level trade estimation results for the benchmark case (i.e., the first approach in which zero trade observations are set equal to one U.S. dollar’s worth) are given in Table 7. Table 7 distinguishes between different versions of the estimated equation. Note that versions B,C,D,E, F, and H are all restricted versions of version A, and version G is a special case of version A. Thus, we can test for those restrictions and decide whether or not they are valid. The test results for these restrictions are given in Table 10. As is evident, all the restrictions are rejected according to our $F$-test results. This suggests that Version A, which is obtained through our model, is selected among all of our versions. This implies that:

- Source specific fixed effects are found to be significant in version B, which supports our assumption of source specific taste parameters in the utility function.

- Trade costs are found to be good specific in version C, which supports our assumption of good specific trade costs.

- Production technology for each good is found to be region specific in version G, which further supports our model.

- Elasticity of substitution across varieties is found to be good specific in version H, which supports our disaggregate level model.
As is evident by Version A in Table 7, the elasticity of substitution across regions is estimated as 5.24 on average. The disaggregate level estimates are given in Table 8. Since the intranational studies within the U.S. such as Wolf (2000), Hillberry and Hummels (2001), and Millimet and Thomas (2007) use gravity equations, they cannot estimate for the elasticity of substitution and the elasticity of distance at the same time. So, we compare our results with the ones in empirical international trade literature and see that our estimates for the elasticity of substitution are lower on average. In particular, Hummel’s (2001) estimates range between 4.79 and 8.26; the estimates of Head and Ries (2001) range between 7.9 and 11.4; the estimate of Baier and Bergstrand (2001) is about 6.4; Harrigan’s (1993) estimates range from 5 to 10; Feenstra’s (1994) estimates range from 3 to 8.4; the estimate by Eaton and Kortum (2002) is about 9.28; the estimates by Romalis (2007) range between 6.2 and 10.9; the (mean) estimates of Broda and Weinstein (2006) range between 4 and 17.3. This difference may be due to the distinction between intranational and international data sets as well as the ignored factors in the literature such as local distribution costs, insurance costs, local taxes and intermediate input trade. Since our model controls for all of these factors, we claim that we have more accurate results intranationally. Someone may claim that the difference between our estimates and the estimates in the literature may also be due to our inclusion of zero trade observations; however, as we will show in our sensitivity analyses, the difference between our estimates and the ones in the literature gets higher when we ignore zero trade observations which is what the studies mentioned above actually do.

Also note that our estimates are highly significant. Moulton (1986) suggests that one should adjust the standard errors for OLS for the fact that the errors are correlated within the groups because of the common group effect. In this context, for robustness, we have also considered Moulton standard errors, and our (t-test) results are almost the same. These results are available upon request. See Moulton (1986) and Donald and Lang (2007) for the details of Moulton standard errors.
According to Version A in Table 7, the distance elasticity is estimated as 0.60 on average. The disaggregate level estimates are given in Table 9. These numbers are higher than the distance elasticity estimates found by the literature, which are about 0.3 (see Hummels, 2001; Limao and Venables, 2001; Anderson and van Wincoop, 2004). This difference is most probably again due to using different frameworks or data sets, as well as due to our inclusion of zero (trade) observations into our analysis. We are going to check for the latter possibility in our sensitivity analysis. Another possible explanation for the difference between our distance elasticity estimates and the ones in the literature may be the mode of transportation for interstate trade. In particular, it may well be the case that the interstate trade is done by air through couriers like UPS, FedEX, and so forth, while the international trade is done in transportation modes different from those. We will also check for this possibility by considering different distance measures in our sensitivity analysis. Another reason may be the usual assumption of iceberg transport costs in the literature. As can be shown, if we had used that assumption instead of having a transportation sector, our distance elasticities would have had lower estimates.\textsuperscript{20} However, since our data set of CFS provides only the income received by the exporter firms (and excludes transportation income), we prefer to distinguish between the exporter income and the transporter income, which is against the iceberg cost assumption.

Although version A (implied by our model) is selected among all estimated versions by our restriction tests, we can still have inference from other versions. Note that versions E and F represent the cases by which we can analyze whether or not there is a home-bias. Again according to Table 7, the values for $\theta_H$ are positive and significant, which according to our definitions for versions E and F, suggest that there is a home-bias across the states.\textsuperscript{20}

\textsuperscript{20}It can be shown easily that the average $\delta (j)$ estimate given in Table 1 (i.e., 0.59) would be replaced by 0.43 under the iceberg cost assumption.
of the U.S. This bias is estimated as 5.73 by Equation E and 2.25 by Equation F. However, since Equation E is a restricted version of Equation F, we can test this restriction. We find that the restriction is rejected, which means that a home-bias of 2.25 is more plausible compared to 5.73. In particular, a typical state has a taste parameter $\theta$ for locally produced goods about 2.25 times more than imported goods. This number is very close to the intranational home-bias estimated by Hillberry and Hummels (2003) which is $exp(0.99) = 2.69$. Therefore, although the literature overestimates the elasticity of substitution measures and underestimates the elasticity of distance measures with respect to our results, the measures of home-bias seem to be similar. One explanation is due to the interaction between the two elasticity measures in Equation III.16. In particular, if two elasticity measures operate in opposite signs (i.e., if one is overestimated and the other is underestimated), then the results for the fixed effects captured by $\theta$ values are not affected too much since two estimation errors cancel each other out to some degree.

Finally, the high adjusted $R^2$ value of 0.42 for Equation A also supports our model. Although version A (implied by our model) is selected among all estimated versions by our significance tests, we can still compare the contribution of each variable in Equation III.16 in explaining interstate trade patterns by considering the adjusted $R^2$ values of each version. In particular, we see that the highest difference of adjusted $R^2$ values takes place between versions A and D&E, which means that source specific fixed effects and good specific trade costs together play an important role in our estimations. The second highest difference of adjusted $R^2$ values takes place between versions A and B&F, which means that source specific fixed effects are significant individually. The third highest difference of adjusted $R^2$ values takes place between versions A and C, which means that good specific trade costs are also significant individually. Finally, the lowest difference of adjusted $R^2$ values takes
place between versions A, G and H, which means that good and region specific technology
parameters and elasticities of substitution across goods, besides the source specific fixed
effects, play a lesser role compared to other parameters, which makes sense since we are
making our analysis within a highly integrated economy, the U.S.

**Sensitivity Analyses**

In order to support our empirical results, in this section, we employ four sensitivity
analyses. The first two are related to zero (trade) observations, the third one is related to
distance measures, and the last one is related to a possible biasedness of the OLS estimator
in log-linearized models.

**Sensitivity Analysis #1** We start our sensitivity analysis by setting zero (trade) obser-
vations equal to one U.S. cent’s worth. In such a case, the estimation results in Table 7 are
replaced by the ones in Table 11. Note that we can again test for the restrictions of versions
B, C, D, E, F, G, and H with respect to version A. The test results for these restrictions
are given in Table 12. As is evident, all the restrictions are again rejected according to our
$F$-test results. This suggests that version A is again selected among all of our equations.
The high adjusted $R^2$ value of 0.40 for Equation A again supports our model.

As is evident by Version A in Table 11, the elasticity of substitution is estimated
as 6.27 on average. The disaggregate level estimates are given again in Table 8. Although
these values are slightly higher than the ones in our benchmark case, they are still lower
than the estimates in the literature on average.$^{21}$

The distance elasticity is estimated as 0.61 on average, which is very close to our
initial estimate in Table 7, yet higher than the ones in the literature. Moreover, the disag-

---

$^{21}$Even if we set zero trade observations equal to 0.01 U.S. cent worth, the elasticity of substitution is estimated as 6.50 on average.
aggreated distance elasticities given in Table 9 are very close to the ones that we estimated initially.

Again according to Table 11, the values for $\theta_H$ are positive and significant, which according to our definitions for versions E and F, suggest that there is a home-bias across the states of the U.S. After a restriction analysis between versions E and F, the restriction in E is again rejected. Thus, a typical state has a taste parameter $\theta$ for locally produced goods about 1.95 times more than imported goods. This number is close to our initial estimate of 2.25.

**Sensitivity Analysis #2** For our second sensitivity analysis, we ignore the zero (trade) observations. In such a case, the estimation results in Table 7 are replaced by the ones in Table 13. We again test for the restrictions of versions B, C, D, E, F, G, and H with respect to version A. The test results for these restrictions are given in Table 14. As is evident, all the restrictions are again rejected according to our $F$-test results. This suggests that version A is again selected among all of our equations. The high adjusted $R^2$ value of 0.60 for Equation A again supports our model.

This time, according to Version A in Table 13, the elasticity of substitution is estimated as 2.70 on average. The disaggregate level estimates are given again in Table 8. These estimates are very low compared to the studies mentioned above even though they also ignore zero trade observations (as in this subsection). We had given possible explanations for this difference above, so we won’t repeat them here.

The distance elasticity is estimated as 0.37 on average. The disaggregate level estimates are again given in Table 9. Although these numbers are closer to the distance elasticity estimates in the literature (that we mentioned above, which are about 0.3), they
are still higher. Thus, the difference between our first two estimates of distance elasticities (i.e., our initial estimate and our first sensitivity analysis) and the estimates in the literature can, to some degree, be explained by the fact that we have included zero (trade) observations in our first two estimations. Nevertheless, the difference doesn’t disappear completely.

According to Table 13, the values for $\theta_H$ are again positive and significant, which according to our definitions for Equations E and F, suggest that there is a home-bias across the states of the U.S. In particular, a typical state has a taste parameter $\theta$ for locally produced goods about 1.93 times more than imported goods after testing for the restriction between Equations E and F and rejecting it. This number is lower compared to our initial estimates and the estimates of Hillberry and Hummels (2003).

**Sensitivity Analysis #3** As we detail in the Appendix, until now, we have used great circle distances instead of actual CFS distances, because average distance measures are not provided by CFS for zero (trade) observations. However, as is shown by Hillberry and Hummels (2001), using great circle distances, instead of actual distances provided by CFS, may overstate the distance measure as in Wolf (2000). In one of our propositions, we had claimed that we already control for this issue by taking the ratio of imports as our dependent variable. Moreover, the coefficient of correlation between the great circle distances and actual distances provided by CFS is calculated as 0.98, after ignoring zero trade observations. Nevertheless, as our third sensitivity analysis, we repeat our sensitivity analysis #2, this time by using the average distance measure provided by CFS instead of the great circle distance measure that we have used until now. In this way, we can compare the effects of great circle distances and the CFS distances on our empirical results.

When we use the CFS distances, the estimation results of sensitivity analysis #2
given in Table 13 are replaced by the ones in Table 15. We again test for the restrictions of versions B, C, D, E, F, G, and H with respect to version A. The test results for these restrictions are given in Table 16. As is evident, all the restrictions are again rejected according to our $F$-test results. This suggests that version A is again selected among all of our equations. The high adjusted $R^2$ value of 0.60 for Equation A again supports our model.

As is evident by Version A in Table 15, the elasticity of substitution is estimated as 2.74 on average. The disaggregate level estimates are given in Table 8. The distance elasticity is estimated as 0.38 on average. The disaggregate level estimates are given in Table 9. All of these estimates are very close to the ones presented for Sensitivity Analysis #2.

According to Table 15, the values for $\theta_H$ are again positive and significant, which according to our definitions for Equations E and F, suggest that there is a home-bias across the states of the U.S. In particular, a typical state has a taste parameter $\theta$ for locally produced goods about 2.03 times more than imported goods after testing for the restriction between Equations E and F, and rejecting it. Although this number is close to our initial estimates, it is slightly higher compared to Table 13.

Overall, if we compare the numbers in Table 13 and Table 15, we see that they don’t change significantly. This result supports our claim that we already control for overstating distances mentioned by Hillberry and Hummels (2001).

**Sensitivity Analysis #4** For our last sensitivity analysis, we repeat our analysis for the benchmark case and the first three sensitivity analyses by using the Poisson Pseudo-Maximum Likelihood (PPML) estimator. As Santos Silva and Tenreyro (2006), and Hender-
son and Millimet (2008) suggest, under heteroskedasticity, the parameters of log-linearized models estimated by OLS may lead to biased estimates; thus, PPML should be used. To show this, Equation III.16 can be written as follows:

\[
\frac{X_{r,a}(j)}{X_{r,b}(j)} = \exp \left( \log \left( \frac{\theta_a}{\theta_b} \right) + (\eta(j) - 1) \log \left( \frac{A_a(j)}{A_b(j)} \right) + \delta(j) \eta(j) \log \left( \frac{D_{r,b}}{D_{r,a}} \right) \right) v_{r,a,b,j}
\]

(III.20)

Assuming \( E[v_{r,a,b,j} | \theta_a, \theta_b, A_a(j), A_b(j), \frac{D_{r,b}}{D_{r,a}}] = 1 \), then Equation III.20 may be estimated consistently using the Poisson Pseudo-Maximum Likelihood (PPML) estimator (see Santos Silva and Tenreyro, 2006). Since Version A has been chosen among all versions earlier, we repeat our analysis only for Version A here. The results are given in Table 17.

As is evident, the (average) elasticity of substitution across varieties \( \eta \) ranges between 2.17 and 2.63, and the (average) elasticity of distance \( \delta \) ranges between 0.30 and 0.59, which are both consistent with our earlier claim that our \( \eta \) estimates are lower than and our \( \delta \) estimates are higher than the ones in the international trade literature.

**Aggregate Level Trade Estimation Results**

The aggregate level trade estimation results are given in Table 18. As we have done for the disaggregate level analysis, we consider four different approaches with two different estimation methods, OLS and PPML. As is evident, the elasticity of substitution across goods is estimated as 1.38 by OLS (2.19 by PPML) in our benchmark case, which is the one that sets zero trade observations equal to one U.S. dollar’s worth. When zero trade observations are set equal to one U.S. cent’s worth, the elasticity of substitution across goods is estimated as 1.27 by OLS (2.19 by PPML). When zero trade observations are ignored,
it is estimated as 1.95 by OLS (2.44 by PPML). Finally, when CFS distance measures are used instead of great circle distances, it is estimated as 1.92 by OLS (2.97 by PPML).

Although the estimates of $\varepsilon$ are lower than the elasticity of substitution across varieties estimates (i.e., $\eta(j)$’s), as expected, according to OLS estimator, they are very close to each other according to PPML estimator. This result is consistent with the view that when goods are aggregated, the elasticity of substitution across them decreases. Nevertheless, these numbers are significantly lower than the estimates in the literature that we have discussed above. As in our disaggregate level analysis, we claim that this difference may be due to distinction between intranational and international data sets as well as the ignored factors in the literature such as local distribution costs, insurance costs, local taxes and intermediate input trade. Since our model controls for all of these factors, we claim that we have more accurate results intranationally. Our results are supported by several sensitivity analyses with high explanatory powers.\textsuperscript{22}

Conclusions

We have written a partial equilibrium model to find motivations for bilateral trade ratios across regions. In particular, we have shown that a region imports more goods from the higher technology regions and fewer goods from the more distant regions, subject to an elasticity of substitution across varieties. Moreover, a region imports more of a good, of which price is lower, subject to an elasticity of substitution across goods. As we have explained in detail in the text, our model has several empirical and analytical benefits compared to the gravity models. Thanks to the disaggregate (state) level data set combined

\textsuperscript{22}We have also tested different restricted versions of Equation III.19, such as common $\gamma$’s or common $P_i(j)$’s, in our aggregate level analysis. We find that none of the restrictions are valid, and therefore Equation III.19 is selected among all versions. These restriction test results are available upon request.
from the Commodity Flow Survey and the U.S. Census Bureau, we are also able to show that our simple model is capable of explaining the interstate trade patterns within the U.S. In particular, we show that the elasticity of substitution measures are overestimated in the literature, while the elasticity of distance measures (thus, trade costs) are underestimated in the literature relative to our estimates.

We have shown that source specific fixed effects and good specific taste parameters are important for bilateral trade patterns, which are usually ignored in the literature. We have also shown that elasticities of substitution across varieties, and trade costs are good specific, which is not a considered fact in most of the aggregate level gravity type studies. Moreover, production technology for each good is found to be region specific rather than country specific. Our sensitivity analyses support our results.

The best strategy for possible future research would be to extend the model of this study toward explaining international trade patterns. Such an analysis would be more convenient with a general equilibrium framework, although a partial equilibrium framework was good enough for this chapter after assuming factor mobility for the production of traded goods.
### Table 7. OLS Estimation Results

<table>
<thead>
<tr>
<th>Equation</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
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<td>(0.08)</td>
<td>(0.16)</td>
<td>(0.06)</td>
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<td>(0.03)</td>
<td>(0.17)</td>
<td>–</td>
<td>(0.03)</td>
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<td>0.59</td>
<td>2.85</td>
<td>2.87</td>
<td>2.78</td>
<td>–</td>
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<td>(0.02)</td>
<td>(0.03)</td>
<td>(0.14)</td>
<td>–</td>
<td>(0.29)</td>
</tr>
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<td>–</td>
<td>$\theta^C$</td>
<td>–</td>
<td>5.73</td>
<td>2.25</td>
<td>$\theta^G$</td>
<td>$\theta^H$</td>
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<td></td>
<td>(·)</td>
<td>–</td>
<td>(·)</td>
<td>–</td>
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<td>(0.22)</td>
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<td>(·)</td>
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<tr>
<td>$\delta\eta$</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>$\eta\delta^G$</td>
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<td></td>
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<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>(·)</td>
<td>–</td>
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</table>

$R$-bar sqd. | 0.42 | 0.31 | 0.37 | 0.26 | 0.26 | 0.31 | 0.41 | 0.41 |

Notes: The standard errors calculated by Delta method are in parenthesis. The sample size for all estimations is 47,819 which is found after considering the independent observations (ratios) and after ignoring the missing observations. The average of the estimated vectors of $\eta^A$ and $\delta^A$ are given in brackets of which full vectors are given in Table 8. The estimated vectors of $\theta^A$, $\theta^C$, $\theta^G$ and $\theta^H$ (all having a size of 505) are omitted to save space. For equations E and F, $\theta$ corresponds to $\theta_H$. For Equations A-F, the estimates for $\delta\eta$ are omitted since the estimates for $\delta$ and $\eta$ are already given separately.
Table 8. Estimated Vectors of Elasticity of Substitution across Varieties

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<tr>
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<th>SA#3</th>
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<td>Other agricultural products</td>
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<td>6.04</td>
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<td>2.45</td>
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<td>(0.06)</td>
<td>(0.07)</td>
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<td>Animal feed, products of animal</td>
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<td>5.99</td>
<td>2.46</td>
<td>2.57</td>
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<td>(0.07)</td>
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<td>Meat, fish, seafood</td>
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<td>Milled grain, bakery products</td>
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<td>Prepared foodstuffs and fats</td>
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<td>Tobacco products</td>
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<td>--------------------------------</td>
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<td>Articles of base metal</td>
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Table 8, continued

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<td>Motorized and other vehicles</td>
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<td>6.69</td>
<td>2.71</td>
<td>2.72</td>
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<td>Transportation equipment</td>
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<td>Furniture, mattresses</td>
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<td>2.73</td>
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<td>Waste and scrap</td>
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Notes: The standard errors calculated by Delta method are in parenthesis. For more details, see 7.

SA#1,2,3 stand for Sensitivity Analysis #1,2,3, respectively.
Table 9. Estimated Vectors of Elasticity of Distance

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<th>SA#3</th>
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<td>0.29</td>
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<td>(0.02)</td>
<td>(0.02)</td>
<td>(0.11)</td>
</tr>
<tr>
<td>Other agricultural products</td>
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<td>0.47</td>
<td>0.27</td>
<td>0.29</td>
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<td>(0.03)</td>
<td>(0.03)</td>
<td>(0.04)</td>
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<td>Animal feed, products of animal</td>
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<td>0.51</td>
<td>0.53</td>
<td>0.46</td>
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<td>(0.06)</td>
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<tr>
<td>Meat, fish, seafood</td>
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<td>0.32</td>
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<td>(0.03)</td>
<td>(0.02)</td>
</tr>
<tr>
<td>Milled grain, bakery products</td>
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<td>(0.03)</td>
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<td>Prepared foodstuffs and fats</td>
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<td>0.41</td>
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<td>Alcoholic beverages</td>
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<td>(0.02)</td>
<td>(0.02)</td>
<td>(0.03)</td>
</tr>
<tr>
<td>Tobacco products</td>
<td>0.23</td>
<td>0.24</td>
<td>0.22</td>
<td>0.27</td>
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<td>Natural sands</td>
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<td>0.55</td>
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<td>(0.03)</td>
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<td>(0.06)</td>
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<td>Gravel and crushed stone</td>
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<td>(0.03)</td>
<td>(0.02)</td>
<td>(0.02)</td>
<td>(0.05)</td>
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104
Table 9, continued

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<th>SA#2</th>
<th>SA#3</th>
</tr>
</thead>
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<tr>
<td>Nonmetallic minerals</td>
<td>0.42</td>
<td>0.45</td>
<td>0.26</td>
<td>0.21</td>
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<tr>
<td>Metallic ores and concentrates</td>
<td>0.16</td>
<td>0.17</td>
<td>0.09</td>
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<td>0.33</td>
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<td>(0.02)</td>
<td>(0.02)</td>
<td>(0.08)</td>
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<tr>
<td>Gasoline, aviation turbine fuel</td>
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<td>0.69</td>
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<td>0.54</td>
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<td>(0.02)</td>
<td>(0.02)</td>
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<td>Coal and petroleum products</td>
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<td>Basic chemicals</td>
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<td>0.27</td>
</tr>
<tr>
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<td>(0.03)</td>
<td>(0.03)</td>
<td>(0.03)</td>
</tr>
<tr>
<td>Pharmaceutical products</td>
<td>0.71</td>
<td>0.72</td>
<td>0.46</td>
<td>0.46</td>
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<tr>
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<td>(0.04)</td>
<td>(0.03)</td>
<td>(0.03)</td>
<td>(0.02)</td>
</tr>
<tr>
<td>Fertilizers</td>
<td>0.37</td>
<td>0.38</td>
<td>0.23</td>
<td>0.16</td>
</tr>
<tr>
<td></td>
<td>(0.03)</td>
<td>(0.03)</td>
<td>(0.03)</td>
<td>(0.06)</td>
</tr>
<tr>
<td>Chemical products</td>
<td>0.80</td>
<td>0.83</td>
<td>0.38</td>
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<td>(0.03)</td>
<td>(0.03)</td>
<td>(0.02)</td>
</tr>
</tbody>
</table>
Table 9, continued

<table>
<thead>
<tr>
<th>Category</th>
<th>Benchmark</th>
<th>SA#1</th>
<th>SA#2</th>
<th>SA#3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plastics and rubber</td>
<td>0.50</td>
<td>0.49</td>
<td>0.38</td>
<td>0.38</td>
</tr>
<tr>
<td></td>
<td>(0.03)</td>
<td>(0.03)</td>
<td>(0.03)</td>
<td>(0.01)</td>
</tr>
<tr>
<td>Wood products</td>
<td>0.89</td>
<td>0.91</td>
<td>0.39</td>
<td>0.39</td>
</tr>
<tr>
<td></td>
<td>(0.03)</td>
<td>(0.02)</td>
<td>(0.02)</td>
<td>(0.02)</td>
</tr>
<tr>
<td>Pulp, newsprint, paper</td>
<td>0.84</td>
<td>0.87</td>
<td>0.32</td>
<td>0.31</td>
</tr>
<tr>
<td></td>
<td>(0.03)</td>
<td>(0.03)</td>
<td>(0.03)</td>
<td>(0.02)</td>
</tr>
<tr>
<td>Paper or paperboard articles</td>
<td>0.99</td>
<td>1.03</td>
<td>0.39</td>
<td>0.39</td>
</tr>
<tr>
<td></td>
<td>(0.03)</td>
<td>(0.03)</td>
<td>(0.03)</td>
<td>(0.02)</td>
</tr>
<tr>
<td>Printed products</td>
<td>0.62</td>
<td>0.63</td>
<td>0.43</td>
<td>0.42</td>
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<tr>
<td></td>
<td>(0.03)</td>
<td>(0.03)</td>
<td>(0.03)</td>
<td>(0.01)</td>
</tr>
<tr>
<td>Textiles, leather</td>
<td>0.34</td>
<td>0.33</td>
<td>0.29</td>
<td>0.29</td>
</tr>
<tr>
<td></td>
<td>(0.03)</td>
<td>(0.03)</td>
<td>(0.03)</td>
<td>(0.01)</td>
</tr>
<tr>
<td>Nonmetallic mineral</td>
<td>0.93</td>
<td>0.96</td>
<td>0.49</td>
<td>0.47</td>
</tr>
<tr>
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<td>(0.03)</td>
<td>(0.03)</td>
<td>(0.02)</td>
</tr>
<tr>
<td>Base metal</td>
<td>0.85</td>
<td>0.87</td>
<td>0.45</td>
<td>0.44</td>
</tr>
<tr>
<td></td>
<td>(0.03)</td>
<td>(0.03)</td>
<td>(0.03)</td>
<td>(0.02)</td>
</tr>
<tr>
<td>Articles of base metal</td>
<td>0.64</td>
<td>0.65</td>
<td>0.40</td>
<td>0.41</td>
</tr>
<tr>
<td></td>
<td>(0.03)</td>
<td>(0.03)</td>
<td>(0.03)</td>
<td>(0.02)</td>
</tr>
<tr>
<td>Machinery</td>
<td>0.42</td>
<td>0.41</td>
<td>0.36</td>
<td>0.35</td>
</tr>
<tr>
<td></td>
<td>(0.03)</td>
<td>(0.03)</td>
<td>(0.03)</td>
<td>(0.01)</td>
</tr>
</tbody>
</table>
Table 9, continued

<table>
<thead>
<tr>
<th></th>
<th>Benchmark</th>
<th>SA#1</th>
<th>SA#2</th>
<th>SA#3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electronic, electrical</td>
<td>0.30</td>
<td>0.28</td>
<td>0.32</td>
<td>0.31</td>
</tr>
<tr>
<td></td>
<td>(0.03)</td>
<td>(0.03)</td>
<td>(0.03)</td>
<td>(0.01)</td>
</tr>
<tr>
<td>Motorized and other vehicles</td>
<td>0.76</td>
<td>0.78</td>
<td>0.37</td>
<td>0.37</td>
</tr>
<tr>
<td></td>
<td>(0.03)</td>
<td>(0.03)</td>
<td>(0.03)</td>
<td>(0.02)</td>
</tr>
<tr>
<td>Transportation equipment</td>
<td>0.33</td>
<td>0.34</td>
<td>0.23</td>
<td>0.22</td>
</tr>
<tr>
<td></td>
<td>(0.04)</td>
<td>(0.04)</td>
<td>(0.04)</td>
<td>(0.03)</td>
</tr>
<tr>
<td>Precision instruments</td>
<td>0.69</td>
<td>0.71</td>
<td>0.25</td>
<td>0.25</td>
</tr>
<tr>
<td></td>
<td>(0.04)</td>
<td>(0.03)</td>
<td>(0.03)</td>
<td>(0.02)</td>
</tr>
<tr>
<td>Furniture, mattresses</td>
<td>0.73</td>
<td>0.75</td>
<td>0.36</td>
<td>0.35</td>
</tr>
<tr>
<td></td>
<td>(0.03)</td>
<td>(0.03)</td>
<td>(0.03)</td>
<td>(0.02)</td>
</tr>
<tr>
<td>Miscellaneous manufactured</td>
<td>0.27</td>
<td>0.25</td>
<td>0.34</td>
<td>0.34</td>
</tr>
<tr>
<td></td>
<td>(0.03)</td>
<td>(0.03)</td>
<td>(0.03)</td>
<td>(0.01)</td>
</tr>
<tr>
<td>Waste and scrap</td>
<td>0.30</td>
<td>0.31</td>
<td>0.40</td>
<td>0.41</td>
</tr>
<tr>
<td></td>
<td>(0.03)</td>
<td>(0.03)</td>
<td>(0.03)</td>
<td>(0.09)</td>
</tr>
<tr>
<td>Mixed freight</td>
<td>1.05</td>
<td>1.05</td>
<td>0.59</td>
<td>0.60</td>
</tr>
<tr>
<td></td>
<td>(0.03)</td>
<td>(0.03)</td>
<td>(0.03)</td>
<td>(0.02)</td>
</tr>
</tbody>
</table>

Notes: The standard errors calculated by Delta method are in parenthesis. For more details, see 7.

SA#1,2,3 stand for Sensitivity Analysis #1,2,3, respectively.
### Table 10. Restriction Test Results

<table>
<thead>
<tr>
<th>Equation</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F$-test</td>
<td>18.74</td>
<td>48.91</td>
<td>23.97</td>
<td>23.86</td>
<td>18.74</td>
<td>7.10</td>
<td>7.29</td>
</tr>
<tr>
<td>d.f. 1</td>
<td>505</td>
<td>74</td>
<td>579</td>
<td>578</td>
<td>504</td>
<td>38</td>
<td>37</td>
</tr>
<tr>
<td>d.f. 2</td>
<td>47,743</td>
<td>47,312</td>
<td>47,817</td>
<td>47,816</td>
<td>47,742</td>
<td>47,276</td>
<td>47,275</td>
</tr>
<tr>
<td>$p$-value</td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
</tr>
</tbody>
</table>

Notes: The null hypothesis is that the restrictions are valid.
Table 11. OLS Estimation Results for Sensitivity Analysis 1

<table>
<thead>
<tr>
<th>Equation</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\eta$</td>
<td>6.27</td>
<td>1.19</td>
<td>6.37</td>
<td>1.15</td>
<td>1.11</td>
<td>1.17</td>
<td>–</td>
<td>6.26</td>
</tr>
<tr>
<td></td>
<td>(0.07)</td>
<td>(0.18)</td>
<td>(0.06)</td>
<td>(0.03)</td>
<td>(0.03)</td>
<td>(0.20)</td>
<td>–</td>
<td>(0.03)</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.61</td>
<td>3.31</td>
<td>0.61</td>
<td>3.41</td>
<td>3.46</td>
<td>3.34</td>
<td>–</td>
<td>0.62</td>
</tr>
<tr>
<td></td>
<td>(0.03)</td>
<td>(0.17)</td>
<td>(0.01)</td>
<td>(0.03)</td>
<td>(0.03)</td>
<td>(0.17)</td>
<td>–</td>
<td>(0.30)</td>
</tr>
<tr>
<td>$\theta$</td>
<td>$\theta^A$</td>
<td>–</td>
<td>$\theta^C$</td>
<td>–</td>
<td>6.00</td>
<td>1.95</td>
<td>$\theta^G$</td>
<td>$\theta^H$</td>
</tr>
<tr>
<td></td>
<td>(·)</td>
<td>–</td>
<td>(·)</td>
<td>–</td>
<td>(0.26)</td>
<td>(0.26)</td>
<td>(·)</td>
<td>(·)</td>
</tr>
<tr>
<td>$\delta\eta$</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>$\eta\delta^G$</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>(·)</td>
<td>–</td>
</tr>
</tbody>
</table>

$R$-bar sqd. | 0.40 | 0.30 | 0.36 | 0.25 | 0.25 | 0.30 | 0.39 | 0.39 |

Notes: The standard errors calculated by Delta method are in parenthesis. The sample size for all estimations is 47,819 which is found after considering the independent observations (ratios) and after ignoring the missing observations. The average of the estimated vectors of $\eta^A$ and $\delta^A$ are given in brackets of which full vectors are given in Table 8. The estimated vectors of $\theta^A$, $\theta^C$, $\theta^G$ and $\theta^H$ (all having a size of 505) are omitted to save space. For equations E and F, $\theta$ corresponds to $\theta_H$. For Equations A-F, the estimates for $\delta\eta$ are omitted since the estimates for $\delta$ and $\eta$ are already given separately.
Table 12. Restriction Test Results for Sensitivity Analysis 1

<table>
<thead>
<tr>
<th>Equation</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
</tr>
</thead>
<tbody>
<tr>
<td>F-test</td>
<td>18.03</td>
<td>46.96</td>
<td>23.06</td>
<td>23.00</td>
<td>18.05</td>
<td>6.91</td>
<td>7.09</td>
</tr>
<tr>
<td>d.f. 1</td>
<td>505</td>
<td>74</td>
<td>579</td>
<td>578</td>
<td>504</td>
<td>38</td>
<td>37</td>
</tr>
<tr>
<td>d.f. 2</td>
<td>47,743</td>
<td>47,312</td>
<td>47,817</td>
<td>47,816</td>
<td>47,742</td>
<td>47,276</td>
<td>47,275</td>
</tr>
<tr>
<td>p-value</td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
</tr>
</tbody>
</table>

Notes: The null hypothesis is that the restrictions are valid.
<table>
<thead>
<tr>
<th>Equation</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\eta)</td>
<td>[2.70]</td>
<td>[1.06]</td>
<td>2.63</td>
<td>1.05</td>
<td>1.01</td>
<td>[0.98]</td>
<td>–</td>
<td>2.68</td>
</tr>
<tr>
<td></td>
<td>[(0.09)]</td>
<td>[(0.18)]</td>
<td>(0.06)</td>
<td>(0.02)</td>
<td>(0.02)</td>
<td>[(0.20)]</td>
<td>–</td>
<td>(0.04)</td>
</tr>
<tr>
<td>(\delta)</td>
<td>[0.37]</td>
<td>[0.92]</td>
<td>0.38</td>
<td>0.86</td>
<td>0.83</td>
<td>[0.93]</td>
<td>–</td>
<td>[0.37]</td>
</tr>
<tr>
<td></td>
<td>[(0.03)]</td>
<td>[(0.02)]</td>
<td>(0.01)</td>
<td>(0.01)</td>
<td>(0.01)</td>
<td>[(0.12)]</td>
<td>–</td>
<td>[(0.18)]</td>
</tr>
<tr>
<td>(\theta)</td>
<td>(\theta^A)</td>
<td>–</td>
<td>(\theta^C)</td>
<td>–</td>
<td>2.03</td>
<td>1.93</td>
<td>(\theta^G)</td>
<td>(\theta^H)</td>
</tr>
<tr>
<td></td>
<td>(\cdot)</td>
<td>–</td>
<td>(\cdot)</td>
<td>–</td>
<td>(0.08)</td>
<td>(0.15)</td>
<td>(\cdot)</td>
<td>(\cdot)</td>
</tr>
<tr>
<td>(\delta \eta)</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>(\eta \delta^G)</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>(\cdot)</td>
<td>–</td>
</tr>
</tbody>
</table>

*R-bar sqd.* | 0.60 | 0.33 | 0.58 | 0.32 | 0.32 | 0.33 | 0.59 | 0.59

Notes: The standard errors calculated by Delta method are in parenthesis. The sample size for all estimations is 12,581 which is found after considering the independent observations (ratios) and after ignoring the missing observations together with zero observations. The average of the estimated vectors of \(\eta^A\) and \(\delta^A\) are given in brackets of which full vectors are given in Table 8. The estimated vectors of \(\theta^A\), \(\theta^C\), \(\theta^G\) and \(\theta^H\) (all having a size of 709) are omitted to save space. For equations E and F, \(\theta\) corresponds to \(\theta^H\). For Equations A-F, the estimates for \(\delta \eta\) are omitted since the estimates for \(\delta\) and \(\eta\) are already given separately.
Table 14. Restriction Test Results for Sensitivity Analysis 2

<table>
<thead>
<tr>
<th>Equation</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
</tr>
</thead>
<tbody>
<tr>
<td>F-test</td>
<td>14.56</td>
<td>7.40</td>
<td>13.89</td>
<td>13.70</td>
<td>14.40</td>
<td>4.61</td>
<td>4.72</td>
</tr>
<tr>
<td>d.f. 1</td>
<td>701</td>
<td>74</td>
<td>775</td>
<td>774</td>
<td>700</td>
<td>38</td>
<td>37</td>
</tr>
<tr>
<td>d.f. 2</td>
<td>12,740</td>
<td>12,113</td>
<td>12,814</td>
<td>12,813</td>
<td>12,739</td>
<td>12,077</td>
<td>12,076</td>
</tr>
<tr>
<td>p-value</td>
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<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
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</table>

Notes: The null hypothesis is that the restrictions are valid.
Table 15. OLS Estimation Results for Sensitivity Analysis 3

<table>
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<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \eta )</td>
<td>[2.74]</td>
<td>[1.08]</td>
<td>2.66</td>
<td>1.06</td>
<td>1.01</td>
<td>[1.01]</td>
<td>( )</td>
<td>2.70</td>
</tr>
<tr>
<td> </td>
<td>[(0.09)]</td>
<td>[(0.18)]</td>
<td>(0.06)</td>
<td>(0.02)</td>
<td>(0.01)</td>
<td>[(0.20)]  </td>
<td>( )  </td>
<td>0.04</td>
</tr>
<tr>
<td>( \delta )</td>
<td>[0.38]</td>
<td>[0.92]</td>
<td>0.38</td>
<td>0.86</td>
<td>0.83</td>
<td>[0.92]</td>
<td>( )</td>
<td>[0.38]</td>
</tr>
<tr>
<td> </td>
<td>[(0.04)]</td>
<td>[(0.12)]</td>
<td>(0.01)</td>
<td>(0.01)</td>
<td>(0.01)</td>
<td>[(0.13)]  </td>
<td>( )  </td>
<td>[(0.18)]</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( \theta )</th>
<th>( \theta^A )</th>
<th>( )</th>
<th>( \theta^C )</th>
<th>( )</th>
<th>2.16</th>
<th>2.03</th>
<th>( \theta^G )</th>
<th>( \theta^H )</th>
</tr>
</thead>
<tbody>
<tr>
<td> </td>
<td>( )  </td>
<td>( )  </td>
<td>( (0.09) )</td>
<td>( (0.19) )  </td>
<td>( )  </td>
<td>( )  </td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( \delta \eta )</th>
<th>( )</th>
<th>( )</th>
<th>( )</th>
<th>( )</th>
<th>( )</th>
<th>( )</th>
<th>( \eta \delta^G )</th>
<th>( )</th>
</tr>
</thead>
<tbody>
<tr>
<td> </td>
<td>( )  </td>
<td>( )  </td>
<td>( )  </td>
<td>( )  </td>
<td>( )  </td>
<td>( )  </td>
<td>( (\cdot) )  </td>
<td>( )  </td>
</tr>
</tbody>
</table>

| \( R \)-bar sqd. | 0.60 | 0.32 | 0.59 | 0.31 | 0.32 | 0.33 | 0.59 | 0.59 |

Notes: The standard errors calculated by Delta method are in parenthesis. The sample size for all estimations is 12,581 which is found after considering the independent observations (ratios) and after ignoring the missing observations together with zero observations. The average of the estimated vectors of \( \eta^A \) and \( \delta^A \) are given in brackets of which full vectors are given in Table 8. The estimated vectors of \( \theta^A \), \( \theta^C \), \( \theta^G \) and \( \theta^H \) (all having a size of 709) are omitted to save space. For equations E and F, \( \theta \) corresponds to \( \theta_H \). For Equations A-F, the estimates for \( \delta \eta \) are omitted since the estimates for \( \delta \) and \( \eta \) are already given separately.
Table 16. Restriction Test Results for Sensitivity Analysis 3

<table>
<thead>
<tr>
<th>Equation</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
</tr>
</thead>
<tbody>
<tr>
<td>d.f. 1</td>
<td>709</td>
<td>74</td>
<td>783</td>
<td>782</td>
<td>708</td>
<td>38</td>
<td>37</td>
</tr>
<tr>
<td>d.f. 2</td>
<td>12,505</td>
<td>11,870</td>
<td>12,579</td>
<td>12,578</td>
<td>12,504</td>
<td>11,834</td>
<td>11,833</td>
</tr>
<tr>
<td>$p$-value</td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
</tr>
</tbody>
</table>

Notes: The null hypothesis is that the restrictions are valid.
### Table 17. PPML Estimation Results

<table>
<thead>
<tr>
<th>Case</th>
<th>$\eta^A$</th>
<th>$\delta^A$</th>
<th>$R$-bar sqd.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Benchmark Case</td>
<td>2.17</td>
<td>0.59</td>
<td>0.54</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td>(0.01)</td>
<td></td>
</tr>
<tr>
<td>Sensitivity Analysis #1</td>
<td>2.17</td>
<td>0.59</td>
<td>0.54</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td>(0.01)</td>
<td></td>
</tr>
<tr>
<td>Sensitivity Analysis #2</td>
<td>2.63</td>
<td>0.30</td>
<td>0.83</td>
</tr>
<tr>
<td></td>
<td>(0.04)</td>
<td>(0.01)</td>
<td></td>
</tr>
<tr>
<td>Sensitivity Analysis #3</td>
<td>2.54</td>
<td>0.34</td>
<td>0.84</td>
</tr>
<tr>
<td></td>
<td>(0.04)</td>
<td>(0.01)</td>
<td></td>
</tr>
</tbody>
</table>

Notes: The standard errors calculated by Delta method are in parenthesis. For each case, the sample size is the same as in the earlier tables. The average of the estimated vectors of $\eta^A$ and $\delta^A$ are presented.
Table 18. Estimation Results for Elasticity of Substitution Across Goods

<table>
<thead>
<tr>
<th></th>
<th>$\varepsilon$</th>
<th>$R$-bar sqd.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Benchmark Case (OLS)</td>
<td>1.38</td>
<td>0.68</td>
</tr>
<tr>
<td></td>
<td>(0.14)</td>
<td></td>
</tr>
<tr>
<td>Sensitivity Analysis #1 (OLS)</td>
<td>1.27</td>
<td>0.68</td>
</tr>
<tr>
<td></td>
<td>(0.13)</td>
<td></td>
</tr>
<tr>
<td>Sensitivity Analysis #2 (OLS)</td>
<td>1.95</td>
<td>0.77</td>
</tr>
<tr>
<td></td>
<td>(0.14)</td>
<td></td>
</tr>
<tr>
<td>Sensitivity Analysis #3 (OLS)</td>
<td>1.92</td>
<td>0.77</td>
</tr>
<tr>
<td></td>
<td>(0.14)</td>
<td></td>
</tr>
<tr>
<td>Benchmark Case (PPML)</td>
<td>2.19</td>
<td>0.13</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td></td>
</tr>
<tr>
<td>Sensitivity Analysis #1 (PPML)</td>
<td>2.19</td>
<td>0.13</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td></td>
</tr>
<tr>
<td>Sensitivity Analysis #2 (PPML)</td>
<td>2.44</td>
<td>0.08</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td></td>
</tr>
<tr>
<td>Sensitivity Analysis #3 (PPML)</td>
<td>2.97</td>
<td>0.07</td>
</tr>
<tr>
<td></td>
<td>(0.02)</td>
<td></td>
</tr>
</tbody>
</table>

Notes: The standard errors are in parenthesis. The sample size for all estimations is 1,319 which is found after considering the independent observations (ratios) and after ignoring the missing observations.
Appendix

This Appendix depicts the details of the data used in the empirical analysis. For the bilateral trade analysis, we use the state-level Commodity Flow Survey (CFS) data obtained from the Bureau of Transportation Statistics for the United States for the year 2002. In particular, we use bilateral interstate trade data for the 2-digit Standard Classification of Transported Goods (SCTG) commodities of which codes are given in Table 19 and of which names are given in the first column of Table 8 in the respective order.

The CFS captures data on shipments originating from select types of business establishments located in all states of the U.S. However, because of data availability, we exclude Alaska, District of Columbia and Hawaii from our analysis. In CFS, shipments traversing the U.S. from a foreign location to another foreign location (e.g., from Canada to Mexico) are not included, nor are shipments from a foreign location to a U.S. location. Shipments that are shipped through a foreign territory with both the origin and destination in the U.S. are included in the CFS data. The mileages calculated for these shipments exclude the international segments (e.g., shipments from New York to Michigan through Canada do not include any mileages for Canada). International export (import) shipments are also included in CFS, with the domestic destination (source) defined as the U.S. port, airport, or border crossing of exit from the U.S.

In order to obtain the technology levels, we first use an approximate crosswalk between 3-digit North American Industry Classification System (NAICS) and 2-digit SCTG obtained from the National Transportation Library of the Bureau of Transportation Statistics. This crosswalk is given in Table 19. After that, we use

\[ A_i(j) = \log \left( \frac{V_i(j)}{P_i L_i(j)} \right) \]

as our measure for the technology levels, where \( V_i(j) \) is the industry/region specific value added; \( P_i \) is the cost of living index for state \( i \) borrowed from Berry et al. (2003); and \( L_i(j) \) is the
industry/region specific hours of labor supplied by the production workers. For the value added of each NAICS industry in each state, we use the state level U.S. Census Bureau data for the relevant industries in 2002.\textsuperscript{23}

For distance measures, we calculate great circle distance between states by using latitudes and longitudes of capital cities of each state published by U.S. Census Bureau. Note that we don’t use the average distance measures given by CFS in our initial analysis, because those measures are available only for realized trade observations. Since we consider zero (trade) observations in our analysis, we use the great circle distance measures that are not included in CFS. Moreover, because we use the ratio of imports of a region (and thus, the ratio of distances), we already control for a possible issue of overstating the distance measures mentioned by Hillberry and Hummels (2001). Nevertheless, we compare our estimation result obtained by great circle distances and with the one obtained by CFS distances in our sensitivity analysis #3 in the text.

\textsuperscript{23}Although we use value added for each industry to calculate technology levels, this should not be necessary the case if we already had a better measure of technology. In other words, our claim in the text saying "We don’t need any income data given the technology levels" still holds. Although the state-level production functions typically include public and private capital, to be consistent with the model, technology is defined on the basis of value added by labor.
Table 19. Crosswalk Between NAICS and SCTG

<table>
<thead>
<tr>
<th>SCTG</th>
<th>NAICS</th>
<th>SCTG</th>
<th>NAICS</th>
<th>SCTG</th>
<th>NAICS</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>311 – 312*</td>
<td>17</td>
<td>324</td>
<td>31</td>
<td>327</td>
</tr>
<tr>
<td>3</td>
<td>311 – 312*</td>
<td>18</td>
<td>324</td>
<td>32</td>
<td>331 – 324*</td>
</tr>
<tr>
<td>4</td>
<td>311 – 312*</td>
<td>19</td>
<td>324 – 325*</td>
<td>33</td>
<td>332</td>
</tr>
<tr>
<td>5</td>
<td>311 – 312*</td>
<td>20</td>
<td>325</td>
<td>34</td>
<td>333</td>
</tr>
<tr>
<td>6</td>
<td>311 – 312*</td>
<td>21</td>
<td>325</td>
<td>35</td>
<td>334 – 335*</td>
</tr>
<tr>
<td>7</td>
<td>311 – 312*</td>
<td>22</td>
<td>325</td>
<td>36</td>
<td>336</td>
</tr>
<tr>
<td>8</td>
<td>311 – 312*</td>
<td>23</td>
<td>325</td>
<td>37</td>
<td>336</td>
</tr>
<tr>
<td>9</td>
<td>311 – 312*</td>
<td>24</td>
<td>326</td>
<td>38</td>
<td>334</td>
</tr>
<tr>
<td>11</td>
<td>212**</td>
<td>26</td>
<td>321</td>
<td>39</td>
<td>337</td>
</tr>
<tr>
<td>12</td>
<td>212**</td>
<td>27</td>
<td>322</td>
<td>40</td>
<td>339</td>
</tr>
<tr>
<td>13</td>
<td>212**</td>
<td>28</td>
<td>322</td>
<td>41</td>
<td>313 – 331*</td>
</tr>
<tr>
<td>14</td>
<td>212**</td>
<td>29</td>
<td>323</td>
<td>43</td>
<td>MIX OF ALL</td>
</tr>
<tr>
<td>15</td>
<td>212**</td>
<td>30</td>
<td>313</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: The source is National Transportation Library of the Bureau of Transportation Statistics. * means that an average of the relevant NAICS industries has been used to obtain technology levels. ** means that there is no corresponding production data for that specific NAICS industry in the U.S. Census Bureau data set; thus, we assume that the technology levels are the same across states for those industries. Finally, SCTG 43 corresponds to mixed freight for which an average of all other NAICS industries in the table are used to obtain technology levels.
CHAPTER IV

A MODEL OF INTERNATIONAL CITIES: IMPLICATIONS FOR REAL EXCHANGE RATES

Introduction

According to the U.S. National Income and Product Accounts, expenditure by consumers at the retail level is about twice what producers receive for the same goods and services. This difference has come to be called the distribution margin. The distribution margin includes transportation costs from the factory gate to the final point of consumption as well as costs and markups at the wholesale and retail stage.

Most existing international models abstract from the distribution sector entirely and focus on the fraction of transportation costs attributable to international shipments. Abstracting from the distribution sector is problematic for three reasons. First, the distribution sector may help us to understand the large and persistent deviations from the Law-of-One-Price (LOP) and Purchasing Power Parity. Second, the general equilibrium interaction of the distribution sector and the production sector is not well understood. Given the prominent role of the dichotomy between traded and non-traded goods in international finance, this is an important omission. Recent evidence also suggests that information technology and scale economies in distribution have altered the efficiency and markup structure of the distribution sector (e.g., the Walmart effect). These developments may have fundamentally altered price dispersion and dynamics, both across locations within countries and across countries. Third, the distribution sector includes the financial, legal, medical and education sectors. These sectors have grown immensely in economic importance over time.
Much of the economic activity engaged by these sectors is geographically segmented due either to the arms-length nature of the exchange, public policy decisions or some combination of the two. Since the shares of expenditure attributable to these sectors tend to rise with the level of development, their economic importance continues to rise globally.

We have two related goals in this paper. The first is to develop a tractable stochastic general equilibrium model of production and distribution at the microeconomic level of individual goods and services across cities. The second is to use the theory to specify a regression model to estimate microeconomic parameters of a cost function, specific to an individual retail good or service, which includes the cost of distribution as well as the more traditional inputs of capital and labor embodied in traded inputs. This cost function is used to conduct a variance decomposition of prices across cities into a distribution margin, a trade cost margin and a residual, good-by-good. The distribution margin is further parsed into the influences of labor and retail infrastructure costs across cities. The Economist Intelligence Unit (EIU) retail price data along with supplementary sources for wages at the city level are used in the empirical work. Since the model assumes perfect competition and abstracts from official barriers to trade, the residual in the regression equation is expected to include markups, official barriers to trade and measurement error.

In the model, each city is inhabited by two representative agents, a manufacturer and a retailer. The manufacturer produces a single homogeneous good using labor as the only input. The manufactured good is shipped to all other cities of the world and deviations in the prices of these traded goods reflect only shipping costs from the factory door to the receiving dock at the retail establishment. The retailer transforms these goods by combining them with her labor and a fixed factor; she may also produce pure services which require no traded inputs at all. The fixed factor is intended to capture retail infrastructure, broadly
defined to include land, buildings, equipment and public infrastructure.

The advantage of drilling down to the level of individual goods and services at the city level is that we can learn a great deal about production structure from the cross-sectional variance in the data. What distinguishes the manufacturing sector from the distribution sector is intimately related to what distinguishes a personal computer from a haircut. Aggregating the data tends to obscure these differences. For example, if trade costs are symmetric, aggregating across imports and exports has the effect of understating their role. Having cities as the locations allows both greater attention to the spatial dimensions of manufacturing specialization and a more precise measure of the distance between production and consumption locations.

We have two sets of results, one for the sources of LOP variance for the median good, the other for the differences in the sources of variance across goods in the cross-section. For the median good in the EIU sample, trade costs account for about 50 percent of LOP deviations, the distribution margin accounts for about 10 percent and the remaining 40 percent is unaccounted for. Because the median good in the EIU has a distribution share of only 0.2, well below the aggregate value of 0.5 in the U.S. National Income and Product Accounts, we also report results centered on this value. Now the tables turn, with distribution costs accounting for 43 percent, trade costs 36 percent, and 21 percent of the variance unexplained.

The relative importance of trade and distribution is fairly stable across sets of locations that include high and low income countries and when comparing within country and cross-border city pairs. The absolute level of cross-sectional variance rises when a border is crossed as one would expect or when comparisons are made between cities with vastly different wealth levels. One exception is the division of the variance accounted for
by the distribution sector into the cost of labor and capital. Variance across low income
countries is dominated by differences in the capital component, with labor playing a small
role. For other countries, the division of the distribution margin across labor and capital is
closer to equality.

Turning to differences in geographic price dispersion across goods, we find substan-
tial heterogeneity consistent with Crucini, Telmer and Zachariadis (2005) who focused on
European Union capital cities using Eurostat micro-price data. The structure of the model
and methodology allow us to say more about the underlying sources of this heterogeneity.
In the international data, the distribution margin accounts for 50 percent of cross-sectional
variance in LOP deviations for the good with the highest distribution share and this fraction
falls to a mere 10 percent as we move to the good with the lowest distribution share. Retail
infrastructure accounts for more than 30 percent of the cross-sectional variance in LOP de-
viations across Canada and the United States for the good with the highest infrastructure
intensity, while accounting for virtually none of the variance for the good with the lowest
infrastructure intensity.

Our theoretical model is closest to Giri (2009) who adds a good-specific distribu-
tion cost to the Eaton-Kortum (2002) model. In Giri’s model distribution services are in
fixed proportion to the physical units of the base good as in Burstein, Neves and Rebelo
(2003) (BNR) with efficiencies drawn from a distribution with a country-specific mean and
common world-wide variance. In contrast, we assume that the technological parameter
for distribution inputs is good–specific while the productivity of the distribution sector is
city-specific. Given that this margin is measurable in the NIPA, we view this as a more
tractable way to model the distribution sector than the random efficiency approach. Our
model shares with Alvarez and Lucas (2007), Atkeson and Burstein (2007), Eaton and Ko-
rtum (2002), and Kanda Naknoi (2008) an interest in the role of traditional trade costs. However, to the extent these papers incorporate a distribution sector, it is a common wedge across all goods in the retail basket, which assumes away any cross-sectional variance in price deviations due to the distribution margin. We find this heterogeneity to be essential for improving our understanding of LOP deviations.

The Model

Each city, indexed by $j$, is inhabited by two representative agents. As is usual in representative agent frameworks, these two agents should be viewed as stand-ins for a large number of atomistic agents of each type, since we will be assuming perfect competition in all factor and final goods markets throughout. One agent specializes in the production of a single traded good, indexed by $i$, while the other specializes in retail trade and production of non-traded services. Production in the manufacturing sector is proportional to labor input, the factor of proportionality is a random productivity variable. Retail production requires both labor and capital. Capital is fixed and is broadly defined to include land, buildings, equipment and public infrastructure. Productivity varies across cities in both the traded goods sector and the retail sector.

Traded goods are subject to iceberg transportation costs which are good and destination specific. Final goods and local inputs (retailer labor and retail capital) are not traded beyond the city limits. While hours and consumption are both choice variables, the assumptions we make in the model imply constant hours in all sectors in all locations, reminiscent of the Long and Plosser (1983) multi-sector, closed economy, real business cycle model. Retail infrastructure, including land, capital and equipment, is in fixed supply (denoted $K_j$).
The good index, \( i \), distinguishes physical objects from the identities of agents and locations only when needed to avoid confusion. In describing the flow of goods from one location to another, the source is the first subscript and the destination is the second subscript. Thus, \( X_{sd} \) refers to the shipment of good \( X \), from city \( s \) to city \( d \). Given the assumption that individuals at each location specialize, \( s \) also indexes the good and the individual to whom the income flows, while \( d \) indicates the expenditure side of the equation. \( \tau_{sd} \) is the iceberg shipping cost from the source to the destination. Since there are no durable goods or assets in the model, adding time subscripts is innocuous: they are omitted here since the focus is on the steady-state properties of the model and long-run deviations from the LOP.

The full solution for quantities and prices is given in the appendix. This section presents the complete model and parts of the equilibrium solution relevant for pricing implications, which is the focus of our empirical work.

**Consumers**

Agents preferences are log-additive over consumption and leisure:

\[
U(C^A_j, L^A_j) \equiv (1 - \theta) \log C^A_j + \theta \log L^A_j, \quad A = m, s.
\]  
(IV.1)

\( C^A_j \) is aggregate consumption and \( L^A_j \) is hours of leisure, for an individual working in city \( j \). There are two individuals in each city, indexed by \( A = m, s \); one is engaged in the manufacture of a single good \( (m) \) and the other is engaged in retailing and service activities \( (s) \).

The consumption aggregate is CES over varieties of manufactured goods produced
worldwide:

\[ C_{ij}^A = \left( \sum_i^{M} (\beta_i)^{\frac{1}{e}} (C_{ij}^A)^{\frac{e-1}{e}} \right)^{\frac{e}{e-1}}. \quad (IV.2) \]

\( C_{ij}^A \) is the consumption of good \( i \) in city \( j \) by worker of type \( A \); \( \varepsilon > 0 \) is the elasticity of substitution across goods, \( \beta_i \) is a good specific taste parameter and \( M \) is the number of manufactured goods in existence. \( M \) is also the number of cities given our specialization assumption.

The two agents inhabiting city \( j \), maximize utility (IV.1) subject to their respective budget constraints:

\[ \sum_i P_{ij} C_{ij}^A \leq W_j^A N_j^A + \varphi^A H_j K_j \quad (IV.3) \]

where \( P_{ij} \) is the price of good \( i \) in destination city \( j \). These prices will be the same for all agents in the same location, but differ across locations for reasons described below. Each of the two residents of city \( j \) earn labor income from their production activities and split the rental income accruing to the retail infrastructure in their city \((\varphi^m + \varphi^s = 1)\), the stock of which is assumed to be fixed at \( K_j \). The rental price of retail infrastructure is denoted \( H_j \).

The consumer’s problem may be solved in two stages. In the first stage, the consumer chooses aggregate consumption and leisure, subject to a budget and time allocation constraint. In the second stage, the consumer minimizes expenditure across goods. Here we collapse the problem to a single stage for brevity. The key equations from the solution
to the consumer’s problem are:

\[ C^A_{ij} = \beta_i \left( \frac{P_{ij}}{P_j} \right)^{-\varepsilon} C^A_j \]  

\[ C^A_j = \frac{W^A_j N^A_j + \varphi^A H_j K_j}{P_j} \]  

\[ N^A_j = 1 - \theta - \varphi^A \theta \frac{H_j K_j}{W^A_j} \]  

\[ L^A_j = \theta + \varphi^A \theta \frac{H_j K_j}{W^A_j} \]

Aggregate real consumption is nominal consumption deflated by the ideal deflator \( P_j \equiv \left( \sum_i \beta_i (P_{ij})^{1-\varepsilon} \right)^{-\varepsilon} \), which ensures \( \sum_i P_{ij} C_{ij} = P_j C_j \) as well as a theoretical mapping from price indices to welfare.

The first equation determines consumption demand for a particular good as a function of the relative price of the good paid by the final consumer in their home market and that individual’s aggregate consumption level. It is important to note that, \( P_{ij} \) is the retail price of good \( i \), in city \( j \); it embodies the cost of local retail services paid to the retailer in addition to the traditional iceberg trade costs of the imported item. The price index, is a weighted average of these retail prices, the closest empirical counterpart would be the CPI index. The second equation is aggregate consumption of an agent, which is equal to her real income. Real income is the sum of nominal wage and rental income, deflated by the local price level, \( P_j \).

The last two equations determine hours of work and leisure. In the absence of rental income, the two agents would work the same number of hours, independent of their relative wage, due to the offsetting income and substitution effect of wages on effort with Cobb-Douglas preferences. In the presence of rental income the requirement for constant effort in equilibrium is that the ratio of rental income to labor income be constant. Most growth models impose restrictions on tastes and technology to ensure constancy of hours.
per capita in the presence of trending productivity.¹

Manufacturers

The production function for manufactured good, \( i \), is:

\[
Y_i = A_i N_i^m
\]  

(IV.8)

where \( A_i \) is the productivity level and \( N_i^m \) is hours of work.

Manufacturers choose labor inputs to maximize profits:

\[
\max_{N_i^m} (Q_{ii} Y_i - W_i^m N_i^m).
\]  

(IV.9)

The manufacturer receives the factor gate price, \( Q_{ii} \), for every unit produced, no matter where the goods end up being sold. Given the assumptions of constant returns to scale, perfect competition and one factor of production, the factory gate price equals the manufacturing wage divided by productivity:

\[
Q_{ii} = \frac{W_i^m}{A_i}.
\]  

(IV.10)

Given specialization, the productivity level in this expression is good and city-specific. The presence of a nation-specific component could easily be incorporated by allowing \( A_i \) to have a common factor across cities located within the same country.

Retailers in each city purchase the manufactured goods and pay a proportional shipping cost. Thus the retail purchase price is the factory-gate price marked up by a proportional shipping cost:

\[
Q_{ij} = (1 + \tau_{ij}) Q_{ii} = (1 + \tau_{ij}) \frac{W_i^m}{A_i}
\]  

(IV.11)

¹Details of these restrictions in the context of the one sector stochastic growth model may be found in King, Plosser and Rebelo (1988).
where $\tau_{ij} > 0$ is the net transportation cost from city $i$ to $j$. $Q_{ij}$ is the price the retailer pays in the destination city. The empirical counterpart to this would be a wholesale price. The local manufacturing plant is close enough to the city to ignore local transportation costs so that $\tau_{ii} = 0$. Effectively this cuts out one intermediary, the wholesaler, and the retailer is viewed as operating next to the factor gate. The destination price of the manufactured good depends: i) positively on both the manufacturing wage and the trade cost; and ii) negatively on manufacturing productivity.

**Retailers**

The retailer in each city optimally chooses how much of each manufactured good to purchase from various cities of the world. The retailer transforms these goods using a fraction of her time endowment and some amount of the local retail infrastructure. The retailer then sells the resulting composite good in the local retail market. The production function for good $i$, sold in city $j$ is:

$$R_{ij} = \left( \left( B_{j} N_{ij}^{s} \right)^{\gamma_{i}} \left( K_{ij} \right)^{1-\gamma_{i}} \right)^{1-\alpha_{i}} \left( G_{ij} \right)^{\alpha_{i}}$$

(IV.12)

$G_{ij}$ is the amount of the manufactured good imported from city $i$, by a retailer in city $j$. $N_{ij}^{s}$ is the fraction of the retailer’s time endowment allocated to the transformation of imported good $i$ for local consumption in city $j$ and $K_{ij}$ is the amount of retail infrastructure allocated to retail good $i$ in city $j$. $B_{j}$ is labor-augmenting productivity specific to the city (equivalently, the retailer), common to all goods sold there.\(^2\)

While the production function is restricted to be common to all locations, it is very flexible across goods. It captures pure labor services (e.g., baby-sitting services) with

\(^2\)In principle one could add good-specific productivity of retailers to account for different levels of competency across goods, but we lack productivity data to operationalize this idea.
\( \alpha_i \) equal to zero and \( \gamma_i \) equal to one; internet purchases (e.g., Amazon.com book purchases),\( \alpha_i \) equal to one, and all points in between.

The retailer in city, \( j \), maximizes profits from the sale of each good, \( i \), by optimally choosing the three inputs needed to produce the good: i) the amount of the traded input, \( G_{ij} \), to import ii) the fraction of her time to devote to the good, \( N_{ij}^s \); and iii) how much local infrastructure to allocate to the activity, \( K_{ij} \):

\[
\max_{N_{ij}^s, K_{ij}, G_{ij}} (P_{ij} R_{ij} - W_j^s N_{ij}^s - H_j K_{ij} - Q_{ij} G_{ij})
\] (IV.13)

At the optimum, the unit price equals marginal cost. Given constant returns to scale and three factors of production, the retail price of good \( i \) sold in location \( j \), is a Cobb-Douglas aggregate of the price (inclusive of trade cost) that the retailer paid to acquire the traded input, \( Q_{ij} \), the retailer’s market wage, \( W_j^s \), and the rental price of retail infrastructure, \( H_j \):

\[
P_{ij} = MC_{ij}^s = \alpha_i \left( (W_j^s / B_j)^{\gamma_i} (H_j)^{(1-\gamma_i)} \right)^{1-\alpha_i} \left( Q_{ij} \right)^{\alpha_i}
\] (IV.14)

\[
\alpha_i \equiv \frac{1}{\alpha_i} \left( (1 - \alpha_i) (\gamma_i)^{\gamma_i} (1 - \gamma_i)^{(1-\gamma_i)} \right)^{1-\alpha_i}
\] (IV.15)

The retail price in city \( j \) is rising in input prices and falling in retail productivity, \( B_j \).

**Equilibrium**

The appendix contains the tedious algebra necessary to arrive at the equilibrium allocations discussed in this section. In the remainder of the paper the consumption aggregator is restricted to Cobb-Douglas to arrive at closed form solutions.

In the global general equilibrium, all the optimality conditions of partial equilibrium must hold for consumers, retailers and manufacturers. In addition, the supply of each
good must equal its total demand, including the resources lost to iceberg shipping costs.

\[ Y_i = \sum_j G_{ij}(1 + \tau_{ij}) \]  

(IV.16)

\[ = \sum_j G_{ij} + \sum_j G_{ij}\tau_{ij} \]  

(IV.17)

\[ = G_i + T_i \]  

(IV.18)

In words: the production of good \( i \), \( Y_i \), is exhausted between the global demand for that good by retailers aggregated across destinations, \( G_i \), and physical loses due to iceberg costs, \( T_i \).

Each individual has a fixed amount of time to devote to hours of work and leisure, here, normalized to unity. The time constraints for the manufacturers and the retailers are thus:

\[ L_j^m + N_j^m = 1 \]  

(IV.19)

\[ L_j^s + \sum_i N_{ij}^s = 1 . \]  

(IV.20)

The summation in the second time constraint reflects the fact that the retailer must divide her time across the \( M \) different retailing activities. The notation implicitly sets the number of goods at the retail level equal to the number of goods in the manufacturing sector. Nothing we derive requires this: we could have some activities that use no traded inputs at all in which case the number of retail goods would exceed the number of manufacturing goods by the number of pure services produced by ‘retailers’ in each city.

The city’s retail infrastructure is exhausted across uses:

\[ K_j = \sum_i K_{ij} . \]
The Data

Our focus is retail price dispersion across international cities at the microeconomic and macroeconomic level. The data source for prices is the Economist Intelligence Unit (EIU) worldwide retail price survey. The survey spans 123 cities, located in 79 countries. Most of the cities are national capitals. The larger number of cities than countries is due to the fact that the survey also includes multiple cities in a few countries. Noteworthy are the 16 U.S. cities included in the survey; the next largest number of cities surveyed equals 5 in Australia, China and Germany. Up to data availability for particular years and cities, the number of goods and services priced is 301. The available sample is 1990 to 2005.

Our goal is to understand the sources of variation in LOP deviations. The Cobb-Douglas functional forms in our model rationalize the use of logarithms of LOP deviations across bilateral city-pairs:

\[ q_{ijk,t} = \ln(S_{jk,t} P_{ij,t}/P_{ik,t}) . \]  

while our long-run focus further suggests the use of time-averaged deviations:

\[ q_{ijk} = T^{-1} \sum_t q_{ijk,t} . \]  

Crucini and Telmer (2007) derive a variance decomposition which is very useful for splitting the total variance of LOP deviations into long-run deviations and time series fluctuations:

\[ Var_{jk,t}(q_{i,jk,t} | i) = Var_{jk}(E_t[q_{i,jk,t} | i, jk]) + E_{jk}[Var_t(q_{i,jk,t} | i, jk)] \]  

\[ V_i = T_i + F_i . \] 

The first term, \( T_i \), which is meant to remind the reader of trade costs broadly defined, is the focus of this paper. It is the variance of the deviations from the LOP across
all location pairs remaining after time-averages of the data have been taken. The role of
time-averaging is to eliminate the time series component of the variation, which is valid
when the data are stationary and sufficiently long time samples are available, which is the
case here. The second term, $F_i$, which is to remind the reader of fluctuations, is the focus
of the international finance literature, often featuring short-run fluctuations of LOP due to
local currency price stickiness. One of the novel findings of Crucini and Telmer is that the
ratio of the variance of the long-run deviations to the total (i.e., $T_i/V_i$) is very large for the
average good: 0.51 for U.S.-Canada intranational pairs and 0.69 for all international city
pairs in the EIU data. In other words, the variance component this model focuses upon is
at least as important in an accounting sense as the focus of business cycle models.

One way to visualize this property of the data is to estimate LOP distributions
using kernel estimation. Figure 18 has eight such kernel estimates. Each chart contains two
lines, one for the distribution of time-averaged LOP, $q_{ijk}$, and one for the distribution of
the time series deviations from the long-run means, $q_{ijk,t} - q_{ijk}$. The upper two charts are
distributions for U.S. city pairs and the lower two are international pairs. The left column
uses non-traded goods prices and the right column uses traded goods prices. The dominance
of the long-run sources of variation relative to the short-run (time series) sources of variation
in most cases is evident in the wider dispersion in the LOP distributions represented by the
solid lines than those represented by the dashed lines in each chart. The role of borders in
increasing price dispersion is evident in comparing the top and bottom panels and the role
of the type of good, as summarized by the classical dichotomy, is apparent by comparing
charts in a particular row across columns.

Table 20 presents summary statistics relating to these figures. The least amount of
price dispersion is found in U.S. traded goods, 0.29 and the greatest amount is found in the
case of non-traded goods involving border crossings, 1.07. More surprising is the fact that non-traded goods in the U.S. have less price dispersion than do traded goods internationally, 0.54 compared to 0.68. Interquartile differences yield similar measures of price dispersion. As discovered by Crucini and Telmer (2007), the time series variation is always less than the long-run variance, with the possible exception of traded goods across U.S. cities and even there one of the two measures (interquartile difference) also gives this ranking. Notice also that the distinction between traded and non-traded goods is obvious in the long-run measure, but ambiguous in the time series measure. Given our emphasis on trade costs, broadly defined and abstraction from stochastic variation due to shocks interacting with sticky prices, this observation is another reason to focus on the time averaged data with our model.

The EIU survey offers little in the way of wage data. Supplemental wage data at the country level come from the International Labor Organization (ILO) survey of occupational and sectoral wages and at the city level from the Union Bank of Switzerland (UBS) survey. The ILO data are averages for countries. They span 49 sectors, 162 occupations and 137 countries. The sample period is annual from 1983 to 2003. The complete list of these sectors, occupations, and countries is found in Oostendorp (2003). In the raw ILO data, the most common period is the month, followed by the hour, but some countries report weekly pay, others give daily rates for some occupations, and so on. In order to have a comparable wage data across countries, the standardized version of ILO survey by Oostendorp (2003) is used: in cases in which the wage data are reported as hourly or daily, then these wages were made (roughly) comparable with monthly wages by multiplication by 160 and 20 respectively. In order to have the largest panel of wage data that are comparable across countries, the monthly wages in US dollars that have been obtained by country-specific and
uniform calibration in Oostendorp (2003) are used.

Wage data at the city level is more appropriate given the EIU retail price data is city based and the intent of the model. International cities were surveyed by the UBS in 2006. These are hourly wages in US dollars, spanning occupations in 71 international cities, 60 of which are also surveyed by the EIU. Among the 60 EIU cities there are four cities from Brazil, Canada, China, France, Italy, Spain and Switzerland; four cities from Germany, and four cities from the U.S. The hourly wages have been obtained by dividing the income per year in each occupation by the city level hours of work in a year, where the latter we collected by a survey, also conducted by the UBS.

Bureau of Labor Statistics (BLS) city wage data from the Occupational Employment Statistics (OES) Survey in 2006 are used to complement UBS data. These wage data are hourly wages in US dollars for the same 16 US cities found in EIU retail price survey. The combination of UBS and BLS wage data, then, provides wage data for 72 EIU cities, comprised of 16 from the BLS and the remainder (non-U.S. cities) from the UBS. Within these 72 EIU cities, in terms of intranational cities, we have two cities from Brazil, Canada, China, France, Italy, Spain and Switzerland; three cities from Germany, and 16 cities from the US.3

In a preliminary part of the analysis, the BLS city wage data are used for broader wage dispersion analysis. These data cover two industries, namely production and sales, for 400 cities (on average) within the U.S. in terms of hourly wages from 1999 to 2006.

A number of trade-offs present themselves in terms of the model focus and the available data. Country-level wage data is generally available for longer periods of time, but fewer locations than city-level wage data. Since the model is explicitly constructed to

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3In an earlier version of this paper we used PWT per capita annual income data covering the annual period from 1990 to 2004 to proxy for real wages. These data span all 79 EIU countries. The results were qualitatively similar to those reported here.
mimic city level aggregation and steady-state features, ideally one would want long time series at the city level. Unfortunately these are simply not available. These trade-offs are discussed as they arise below.

Land prices and rents are even more difficult to come by than are wages and prices. We use the EIU survey data item: “Typical annual gross rent for top-quality units, 2,000 square meters, suitable for warehousing or factory use.”

The other two pieces of information are sectoral estimates of the distribution shares, $1 - \alpha$, which are calculated from a combination of U.S. NIPA data and input-output tables. The NIPA data extend to 57 sectors, while the input-output data span 33 sectors. The NIPA shares are computed as the value the producers receive relative to the value consumers pay for the output of a particular sector. The distribution margin, $1 - \alpha$, includes transportation costs, retail and distribution costs and markups. Sectors involving arms-length transactions, such as medical services are recorded in the NIPA as though the producer and consumer valuation is equal. While this is literally true in some cases, this accounting fails to distinguish local inputs from traded inputs used in the production of services. For these sectors we use the input-output tables to determine the distribution share. These sectoral measures from the NIPA complement the good-level parameters estimated using a regression framework discussed below.

Finally, the greater circle distance between cities in the EIU sample is used to

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4 One additional commercial rental price is available in the EIU, “Typical annual gross rent for a 1,000 square meter unit in a Class A building in a prime location.” Results are very similar with this alternative measure.
estimate the trade cost component of the LOP deviations at the retail level.\(^5\)

**Microeconomic Sources of Long-run Variation in Wages**

In the model, wage deviations arise across the retail and manufacturing sectors and across cities. The amount of labor income accruing to the manufacturer relative to the retailer in city \(j\) is,

\[
\frac{N^m_j W^m_j}{N^s_j W^s_j} = \frac{\sum_i \alpha_i \beta_i}{\sum_i (1 - \alpha_i) \gamma_i \beta_i} \tag{IV.25}
\]

Which is intuitive: the numerator is an expenditure share weighted average of labor’s share of manufacturing and the denominator is the counterpart in retailing. The appearance of the parameter \(\gamma_i\) in the denominator accounts for the fact that retail production involves some retail infrastructure, unless \(\gamma_i = 1\), in which case retail production is labor-only. Note, also, that the ratio is the same in all cities.

As the primary interest is wage variation across cities as an explanation for cost and price variation across cities, we would like to understand the wage ratio and effort ratios separately. The equilibrium relative *sectoral* wage is given by:

\[
\frac{W^m_j}{W^s_j} = \frac{\varphi^m (1 - \theta) - N^s_j}{1 - \varphi^m (1 - \theta) - N^m_j}
\]

Thus, given fixed shares of rental income across agents in the city, relative wages and relative hours move inversely as one would expect. The appendix shows that the equilibrium effort

\(^5\)Hummels (2001) provides the most comprehensive estimates of sectoral trade costs using import unit values, a more direct method than employed here. Unfortunately these estimates are available for a limited number of countries and are more aggregated than our retail data.
levels are:

\[ N_j^m = \frac{(1 - \phi_0)(1 - \theta)}{(1 - \phi_0) + \varphi^m \theta (\phi_0 - \phi_1)} \]

\[ N_j^s = \frac{\phi_1 (1 - \theta)}{\phi_1 + \varphi^s \theta (\phi_0 - \phi_1)} \]

where \( \phi_0 \equiv \sum_i (1 - \alpha_i) \beta_i \) and \( \phi_1 \equiv \sum_i (1 - \alpha_i) \gamma_i \beta_i \). Effort in both sectors is declining in the share of rental income allocated to the agent (a wealth effect), and in the preference for leisure (\( \theta \)), as one would expect.

Substituting these expressions into the wage ratio leads to the following expression for relative wages:

\[ \frac{W_j^m}{W_j^s} = 1 - \frac{\varphi^m (1 - \phi_0) + \varphi^A \mu}{\varphi^m \phi_1 + (1 - \varphi^A) \mu} \]

\[ \mu = \theta (\phi_0 - \phi_1). \]

As the retail sector becomes more labor intensive (thus reducing rental income), \( (\phi_0 - \phi_1) \) converges to zero and the model reverts to the labor-only version with a common fraction of available hours worked by both agents, equal to \( (1 - \theta) \) and the sectoral wage ratio converges to:

\[ \frac{W_j^m}{W_j^s} = \frac{1 - \phi_0}{\phi_0} = \frac{\sum \alpha_i \beta_i}{\sum_i (1 - \alpha_i) \beta_i} \]

which is exactly the same expression as labor \textit{income} shares in the more general case (see equation (IV.25)).

Turning to wage differences \textit{across} cities things are much simpler even in the
general case:

\[
\frac{W_j^s}{W_k^s} = \frac{\alpha_j \beta_j}{\alpha_k \beta_k}
\]  (IV.26)

\[
\frac{W_j^m}{W_k^m} = \frac{\alpha_j \beta_j}{\alpha_k \beta_k}
\]  (IV.27)

The cross-city wage differential is the same in both sectors and is determined by the product of the taste and technology parameters in the two locations being compared. The intuition for this result is as follows. Consider, first, the special case in which all goods use traded inputs in the same proportion, \(\alpha_j = \alpha\). Wages are higher in locations that produce the goods most preferred by consumers, given by the comparison of \(\beta_j\) and \(\beta_k\), a demand-side effect. Next, consider the case with symmetric tastes across goods \(\beta_j = \beta\); then wages are the highest for producers of manufactures requiring the least amount of input from retailers (i.e., the lowest \(1 - \alpha\)). Essentially, the higher the distribution share, the less productive is an hour allocated to production of the manufactured good in terms of delivering a unit of consumption to final consumers. This lowers the equilibrium real wage.

Wage data are available by occupation or sector of employment. Our model focuses on the distinction between goods and services, suggesting the production sector definition is more appropriate. However, we use both labor classifications as a robustness check.

The more comprehensive of the sources used is the ILO survey of wage levels across countries. These data span 49 sectors, 162 occupations and 137 countries. The sample period is annual from 1983 to 2003.\(^6\) Because the model is intended to be based on city-level data, the preferred measure is wage data from the UBS that span 14 occupations and 71 international cities for the year 2006.

According to the model, if the retail sector uses only labor and traded goods,

\(^6\)Useful technical documentation is found in Remco H. Oostendorp (2003).
the ratio of manufacturing wages to service wages provides an estimate of the overall scale of
the distribution sector \( \frac{W^m_j}{W^s_j} = \frac{\kappa}{1 - \kappa} \), \( \kappa = \sum_i \beta_i \alpha_i \). Since we lack consumption expenditure shares at the present time, we associate this with the distribution share alone since using the symmetric taste version of the model we have: \( \kappa = \bar{\alpha} \). A direct way to measure the overall size of the distribution sector is to use U.S. NIPA data and input-output data. Crucini and Shintani (2008) do exactly this and find \( \kappa = 0.57 \). The advantage of their calculation is that it is based on expenditure weighting of sectoral \( \alpha \)'s.

Table 21 reports the sectoral wage ratio averaged across locations as well as the implied value for \( \kappa \). It turns out that the direct and indirect (model-based) estimates are equal when U.S. wages in production sector relative to the sales sector are used. The wage ratio in the international data is consistent with a value \( \kappa \) of 0.52. While this is a modest difference from the U.S. value, the implied manufacturing wage premium is quite dramatically affected: it is a factor of 5 smaller than the U.S. case. It could be that relative productivity differences are the cause. Another possibility is that the U.S. and international agencies have different classification systems for the sectors.

As the theory is a two-sector model, any sectoral variation in wages in a particular city is attributable to wage differences across the manufacturing and service sectors. Variation in wages across sub-sectors are abstracted from entirely. Thus, it is important for the theory that wages differ significantly across locations and less so across sectors other than the two sectors emphasized by the model (retail and manufacturing). And this is what is found.

Table 22 conducts a variance decomposition by sector and country for time-averaged wages in the case of the ILO survey and an analogous decomposition by sector and city for wages in 2006 for the UBS survey data. Since the answer may depend on the set
of sectors and locations used, we consider three location groups and allow for many sectors throughout. The location groups are the entire world, the OECD and the LDC.

Based on the ILO wage data: locations account for between 72 and 85 percent of the cross-sectional variation in wages, sectoral differences account for less than 10 percent. The dominance of location in accounting for wage dispersion is somewhat less pronounced when the data is organized by occupation: location effects drop to between 38 and 65 percent. Most of difference is not attributed to a pure sectoral component, but rather an interaction of location and sector. The UBS tell a similar story to the ILO for location effects, with the occupation effect rising in contribution due to a lower interaction with location compared to the ILO.

In sum, location is a key component of wage dispersion with the precise fractions depending somewhat on the set of locations examined and the precise definition of wage categories.

**Microeconomic Sources of Long-run Variation in Real Exchange Rates**

We turn, now, to the main focus, price dispersion. In the model, prices consumers actually pay may differ from factory gate prices for two reasons. The first is the trade cost to import the good from the foreign production location. The second is the value added by the retailer. To simplify the notation, all international prices have been converted to common currency units (it does not matter which numeraire is chosen). The ratio of the price of good $i$ in city $j$ relative to $k$, based on the theory is:

$$\frac{P_{ij}}{P_{ik}} = \left( \frac{W_{ij}^s/B_j}{W_{ik}^s/B_k} \right)^{\gamma_i(1-\alpha_i)} \left( \frac{H_j}{H_k} \right)^{(1-\gamma_i)(1-\alpha_i)} \left( \frac{Q_{ij}}{Q_{ik}} \right)^{\alpha_i}.$$

Noting that the last term reduces to the ratio of trade costs from the single source of good $i$
to each of the destinations, \( j \) and \( k \) and taking logs, defines the Law-of-One-Price deviation across a city pair:

\[
q_{ijk} = (1 - \alpha_i) [\gamma_i \omega_{jk} + (1 - \gamma_i) h_{jk}] + \alpha_i \tau_{ijk} \tag{IV.28}
\]

\[
= \rho_i \omega_{jk} + \chi_i h_{jk} + \alpha_i \tau_{ijk}.
\]

The retail margin is the first term in square braces; it is a weighted average of the productivity-adjusted wage and the rental price differential faced by retailers in the two cities. The weights attached to the relative input prices in the retail sector depend on \( \gamma_i \). The entire retail component gets weighted by its overall share in the production of the final good, \((1 - \alpha_i)\). The second term is the relative trade cost. The last line, used to specify our regression approach, expresses the relationship in terms of the three key cost ratios, retail wages, rental prices and trade costs.

The aggregate real exchange rate in our theory follows directly from equation (IV.28) since the consumption aggregator is Cobb-Douglas (i.e., \( \varepsilon = 1 \)):

\[
q_{jk} = \rho \omega_{jk} + \chi h_{jk} + \sum_i \beta_i \alpha_i \tau_{ijk} \tag{IV.29}
\]

\[
= \sum_i \beta_i \gamma_i (1 - \alpha_i), \quad \chi \equiv \sum_i \beta_i (1 - \gamma_i)(1 - \alpha_i)
\]

The aggregate real exchange rate has a number of interesting features. The distribution component of the PPP deviations are driven by exactly the same wage and rental price differentials as was true of the LOP deviations, the impact factors are consumption-expenditure-weighted production parameters, \( \rho \) and \( \chi \). The trade cost component is more convoluted because the expenditure shares, production coefficients, and trade costs are good specific. However, it seems plausible that the individual deviations could average out across goods.
since the $\tau_{ijk}$ are expected to vary in sign across goods.

**Regression Specification**

This section conducts a variance decomposition of retail prices into the channels described by the equilibrium model. Adding a measurement error term to the theoretical equation for the LOP deviation, gives:

$$q_{ijk} = \rho_i \omega_{jk} + \chi_i h_{jk} + \alpha_i \tau_{ijk} + \varepsilon_{ijk} \tag{IV.30}$$

Data on retail prices, wages and rent, are available, but no data on retail productivity or trade costs exist for this cross-section of locations, at this level of disaggregation. The raw wage ratios are used in place of $\omega_{jk}$ and a two-stage estimation approach is used to infer the impact of trade costs.

The first-stage regression is:

$$q_{ijk} = \rho_{1i} \omega_{jk} + \chi_{1i} h_{jk} + \theta_{ijk}. \tag{IV.31}$$

where $\theta_{ijk}$ is an estimated residual, which, according to the theory, is the LOP deviation in the traded component of cost. In practice it will incorporate other sources of deviations as well. In an attempt to purge these other factors from the pure trade cost component, the estimated residuals are projected on bilateral distances. To accomplish this, define the direction-of-trade indicator function:

$$I_{ijk} = \begin{cases} 
1 & \text{if } \theta_{ijk} > 0 \\
-1 & \text{if } \theta_{ijk} < 0 
\end{cases} \tag{IV.32}$$

where $\theta_{ijk} = q_{ijk} - \rho_{1i} \omega_{jk} - \chi_{1i} h_{jk}$ from the first-stage regression. In words: imports (exports) are assumed to be relatively expensive (inexpensive) at the destination (source).
Consider, now, the more elaborate equation for stage two:

\[ q_{ijk} = \rho_{2i}\omega_{jk} + \chi_{2i}h_{jk} + \varsigma_{2i}I_{ijk}d_{jk} + \varepsilon_{ijk} \]  \hspace{1cm} (IV.33)

\[ \rho_{2i} = (1 - \alpha_i)\gamma_i \]  \hspace{1cm} (IV.34)

\[ \chi_{2i} = (1 - \alpha_i)(1 - \gamma_i) \]  \hspace{1cm} (IV.35)

\[ \varsigma_{2i} = \alpha_i\delta_i \]  \hspace{1cm} (IV.36)

with the trade cost replaced by \( I_{ijk}\delta_i d_{jk} \). The indicator function ensures the sign of the implied trade cost is consistent with the sign of the residual estimated in stage one. The greatest circle distance between locations \( j \) and \( k \) is the empirical counterpart to \( d_{jk} \) and goods are allowed to have different trade cost elasticities with respect to distance, \( \delta_i \). The benefit of projecting the prices on wages, rents, and the indicator function multiplying distance is that we relegate any sources of variation in retail prices not correlated with wages, rental prices or distance to the error term. This gives us more confidence that the wage, rental, and trade cost components are capturing what the model says they should.

The model is best suited to describe the long-run properties of real exchange rates since we abstract from nominal exchange rate variation and sticky prices. While we have a long panel of EIU retail price data from which to construct time-averages and target long-run price dispersion, as noted earlier, we lack comparable city-level panel data on wages. Moreover, the argument could be made for estimating the parameters with a single cross-section. Our benchmark estimation and variance decomposition uses time-average data as available (i.e., for \( q_{ijk} \) and \( h_{ijk} \)) and wage data for a single cross-section in 2006. Wage data from the UBS is used for cities outside of the U.S. and wage data from the BLS is used for U.S. cities. Preliminary experimentation with alternatives does not seem to alter the main thrust of the results.
We see in Table 23, that the empirical model captures the majority of long-run retail price dispersion across locations for all groupings of the data. The range of variance accounted for is between 70 percent and 90 percent for the median good when pooling all international cities or just those in North America. The fit of the model is excellent over much of the distribution of goods. The lowest quartile for the $R^2$ is a very respectable 0.67 (the OECD cross-border pairs). In summary, the empirical model fits well across sub-set of locations and across goods ranging from haircuts to personal computers.

**Variance Decomposition**

Using the estimated equations motivated by the theory, we are able to provide a cross-sectional variance decomposition analysis according to the following equation (we suppress the residual and covariance terms here for expositional convenience; also the parameters used in computations will be those from the second stage estimation, though we suppress the subscript denoting this as well in what follows):

$$\text{var}_{jk}(q_{ijk}) = (1 - \alpha_i)^2 D_{ijk} + \alpha_i^2 d_{ijk}.$$ 

According to the theory, geographic price dispersion at the level of an individual good, $i$, is a weighted average of the geographic dispersion in distribution costs, $D_{ijk}$, and the geographic dispersion of destination prices for traded inputs, $d_{ijk}$. The relative contribution of distribution costs and trade costs for a particular good hinges on the value taken by the distribution share, $\alpha_i$, ranging from close to zero for a personal computer to close to 1 for a haircut.

Recall that the distribution cost component is a weighted average of the dispersion
in wages and rental prices:

\[ D_{ijk} \equiv \left[ \gamma_i^2 \text{var}_{jk}(\omega_{jk}) + (1 - \gamma_i)^2 \text{var}_{jk}(h_{jk}) \right]. \]

Finally, the quantitative role of trade costs depends on the relationship between trade costs and distance interacting with the direction of trade:

\[ d_{ijk} = \delta_i^2 \text{var}_{jk} [I_{ijk} d_{jk}]. \]

Table 24 presents estimates of the variances of retail prices, wages and rental prices for various location groups: i) intranational city pairs (which given the data, is dominated by U.S. city pairs), and ii) cross-border city pairs (using three groupings, OECD, LDC and World).

The conventional wisdom is that factor markets are close to perfectly integrated intranationally, while the immobility of labor and possibly capital prevents this from occurring internationally. This seems to be a reasonable assumption of labor markets since we find wage dispersion of 3 or 4 percent, for intranational pairs. It appears not to be true of rental prices, where dispersion is about 30 percent. These numbers are fairly robust of inclusion of intranational city pairs outside of North America.

Turning to cross-border city pairs, consistent with expectations, we see less of a tendency toward factor-price equalization than within countries. In fact, there is an approximate tripling of the variance of wages as a consequence of crossing the U.S.-Canadian border. The border width appears less dramatic when we look at rental prices, where the variance merely doubles. When we expand the set of international comparisons to the OECD, we find virtually no impact on wage dispersion, but a large impact on rental price dispersion. Expanding the geography further to include both the OECD and non-
OECD (the row labelled WORLD), wage dispersion increases considerably more than rental price dispersion. The main implication for retail price dispersion, though, is that factor price dispersion rises by a factor of about 30 for both wages and rental prices as we move from intranational city pairs to international city pairs. Distribution costs, therefore, are expected to be significant contributors to the absolute level of LOP deviations at the retail level, particularly for cross-border pairs since factor prices are far from being equalized internationally. Moreover, the relative contribution of distribution costs relative to trade costs will shift across goods according to the distribution share parameter, $\alpha_i$.

We turn now to the details of the variance decomposition. The analysis considers both a variance decomposition for the median good and results good-by-good. In each case we contrast interesting geographic groups. For the discussion that follows, it is useful to refer to the full variance decomposition:

$$\text{var}_{jk}(q_{ijk}) = \underbrace{[(1 - \alpha_i)\gamma_i]^2\text{var}_{jk}(\omega_{jk}) + [(1 - \alpha_i)(1 - \gamma_i)]^2\text{var}_{jk}(h_{jk})}_{\text{var}_{jk}(\gamma_{ijk})} + \underbrace{\alpha_i\delta_i^2\text{var}_{jk}[I_{ijk}d_{jk}] + \text{var}_{jk}[\varepsilon_{ijk}]}_{\text{cov terms}}$$

Consider a good which uses no traded inputs at the retail level ($\alpha_i = 0$). The prediction simplifies reduces to:

$$\text{var}_{jk}(q_{ijk}) = \gamma_i^2\text{var}_{jk}(\omega_{jk}) + (1 - \gamma_i)^2\text{var}_{jk}(h_{jk}) + \text{cov terms}$$

We key insight here, is that price dispersion is entirely due to retail costs associated with wage and rental price dispersion, $\text{var}_{jk}(\omega_{jk})$ and $\text{var}_{jk}(h_{jk})$, respectively. These numbers naturally depend on the locations pooled in the estimation for the reasons discussed earlier. Borders matter.

At the opposite end of the continuum is a good with no retail costs at all (e.g., a
good available on the internet that trades up to a shipping cost everywhere in the world \( (\alpha_i = 1) \). Now the expression for the predicted price dispersion reduces to:

\[
\text{var}_{jk}(q_{ijk}) = \delta_i^2 \text{var}_{jk}[I_{ijk}d_{jk}]
\]

This is an intriguing expression. The coefficient out front is the elasticity of trade cost with respect to distance (recall, the empirical model assumes a log-linear proportional trade cost function as is typical in the gravity literature). The variance of distance is a function of the set of locations under examination. As bilateral distance become less symmetric (less equal), trade cost matters more for price deviations.

The variance decomposition results are given in Table 25. For the median good, distribution costs account for between 5 and 20 percent of overall price dispersion, depending on the locations used. The wage component tends to account for more of this dispersion than the rental component. An exception is the LDC group where the rental component accounts for 12.6 percent of the dispersion, compared to only 2.5 percent for wages. Trade costs dominate the picture throughout the table, accounting for as much as 60 percent of the price dispersion for cross-border OECD pairs, to a lower, but still very substantial, 36.1 percent across the Canada-U.S. border. Approximately 30 percent of the variance is left unaccounted for by the model. This variation could be due to a combination of markup variation, official barriers to trade or measurement error. The covariance across effects is typically less that 5 percent. The bottom line of the analysis of the median good are that trade costs dominate independent of the location or border crossing and that distribution margins are important enough not to ignore.

Variation across goods within the cross-section, is interesting. Figure 19 shows the variance decomposition at the individual good level as a function of the traded input
share, $\alpha_i$. To make these easier to read we have smoothed the profiles by taking centered moving averages of the variance share across 10 goods. Starting with all international cross-border pairs and the good with the lowest traded input share (roughly 0.4), wage dispersion accounts for about 45 percent of price dispersion. As we move to goods with the highest traded input share (roughly 0.97), wage dispersion accounts for almost none of the price dispersion. Of course if this good had literally no non-traded inputs the contribution would necessarily be exactly zero. The OECD group tells a similar story with about 30 percent of price dispersion accounted for by wage dispersion at one end of the continuum of goods and less than 10 percent contributed for goods embodying mostly traded inputs. The Canada-U.S. pairs have a lower contribution from wage dispersion as we would expect given the similar wage levels of the two countries, the contribution of this component also declines as $\alpha$ rises, though not as smoothly as the other groups. In most cases, the falling contribution of wage differences is associated with a rising role for trade costs. The intranational pairs show less heterogeneity in the proportion of variance explained by various components as the trade share of final good production varies. Partly this reflects the lower variance of wages and rent across cities within countries. Nonetheless, the contribution of distribution costs is not negligible for the intranational pairs either.

Figure 20 displays the same variance decomposition by good plotted against the labor share of total retail cost, $\gamma_i$. We see the dramatic effect of this parameter on the split of distribution margin variance across labor and rent. As we move across goods based on this parameter, the contribution of rent goes from zero to about 40 percent in the Canada-U.S. panel and from zero to about 20 percent in the world grouping (for cross-border city pairs). The contribution of wage dispersion tends to follow the same pattern in reverse, maintaining the total share of price dispersion due to distribution costs. The OECD is anomalous in
the sense that the distribution share contributes about 10 percent without much variation across goods until we reach very high labor intensities in distribution. Turning to the intranational pairs, the overall contribution of wage dispersion is rising in its cost share as one would expect.

The results for the median good in the EIU cross-section seem to downplay the role of distribution costs relative to trade costs. Given the dramatic differences in how the variance decomposition plays out across goods, the natural question that arises is how representative the EIU sample is of the CPI basket. A second issue is the extent to which the estimated distribution share matches up with the direct measures in the NIPA data.

Regarding the second issue, the average estimated value of the distribution share across goods we use in the estimation is 0.2. This value is significantly below, 0.5, the average we get when we merge our micro-data with the U.S. NIPA and use the sectoral values of the distribution share from that source. Moreover, the difference between the regression estimates of the distribution share and the direct NIPA measure is not due to a few outliers: 151 out of 160 regression coefficients values are below their NIPA counterparts. This suggests that our good-level estimates of the distribution shares are downward biased.

To account for this estimation bias and make the results relevant for aggregate consumption, we recompute our variance decomposition using goods with distribution shares in the neighborhood of $\alpha = 0.5$, the expenditure weighted average of the distribution shares found in the U.S. NIPA data. What we do is average the decomposition results across 5 goods on either side of this value. Table 26 reports these findings. We see that the contribution of the distribution margin is much more significant. Wage dispersion alone now accounts for more than one-third of retail price dispersion when all cross-border city pairs are pooled (WORLD). The role of wages for the OECD and LDC groupings is more
limited suggesting the city pairs that straddle high and low income countries are the reason for the much elevated wage component. It is interesting to note that for the Canada-U.S. pairs, wage dispersion plays a significant role as well. Keep in mind, however, that the absolute dispersion of prices across North American cities is about one-fifth of that existing across cities of the world, thus the significant role of wage dispersion in North America is partly due to the fact that there is little in the way of price dispersion to explain in North America relative to the broader international sample.

Conclusions

Consumers face prices that are to a varying degree, location-specific. Our model of production and distribution across cities shows how these differences are shaped by the distances separating cities due to trade costs, the good-specific share of retail distribution and its division between local labor and rental costs. While we found trade costs dominated distribution costs by a factor of 5 to 1 for the median good in the sample, their relative contribution varies greatly across goods. For final goods that involve mostly non-traded inputs, distribution margins dominate trade costs. Given that most of the goods in the EIU have low distribution shares, these unweighted averages understate the role of distribution margins in the aggregate consumption basket. Using the aggregate distribution share and estimates of the variance decomposition for individual goods with that share, the tables are turn: distribution costs now clearly dominate trade costs.

In future work we will undertake analysis of PPP using our model and empirical methodology. We expect the distribution margin will dominate trade costs in this case we well. These findings point to the importance of incorporating a distribution sector into existing international trade and macroeconomic models.
Table 20. Kernel Density Summary Results

<table>
<thead>
<tr>
<th></th>
<th>Long-run LOP deviations</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Standard</td>
<td>Interquartile</td>
<td>First</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Deviation</td>
<td>Range</td>
<td>Quartile</td>
</tr>
<tr>
<td>U.S. cities</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Traded goods</td>
<td>0.294</td>
<td>0.385</td>
<td>-0.161</td>
<td>0.224</td>
</tr>
<tr>
<td>Non-traded goods</td>
<td>0.543</td>
<td>0.616</td>
<td>-0.250</td>
<td>0.366</td>
</tr>
<tr>
<td>International cities</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Traded goods</td>
<td>0.681</td>
<td>0.796</td>
<td>-0.365</td>
<td>0.431</td>
</tr>
<tr>
<td>Non-traded goods</td>
<td>1.069</td>
<td>1.092</td>
<td>-0.497</td>
<td>0.595</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Short-run LOP deviations</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Standard</td>
<td>Interquartile</td>
<td>First</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Deviation</td>
<td>Range</td>
<td>Quartile</td>
</tr>
<tr>
<td>U.S. cities</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Traded goods</td>
<td>0.250</td>
<td>0.295</td>
<td>-0.151</td>
<td>0.144</td>
</tr>
<tr>
<td>Non-traded goods</td>
<td>0.258</td>
<td>0.295</td>
<td>-0.151</td>
<td>0.144</td>
</tr>
<tr>
<td>International cities</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Traded goods</td>
<td>0.412</td>
<td>0.417</td>
<td>-0.209</td>
<td>0.209</td>
</tr>
<tr>
<td>Non-traded goods</td>
<td>0.488</td>
<td>0.430</td>
<td>-0.215</td>
<td>0.215</td>
</tr>
</tbody>
</table>

Note: Long-run LOP deviations are time-averaged LOP deviations, short-run LOP deviations are the difference between the raw LOP series and the long-run means.
Table 21. Mean Sectoral Wage Differentials

<table>
<thead>
<tr>
<th></th>
<th>$W_j^m/W_j^s$</th>
<th>Implied $\kappa$</th>
</tr>
</thead>
<tbody>
<tr>
<td>World Manufacture to Sales (ILO)</td>
<td>1.07</td>
<td>0.52</td>
</tr>
<tr>
<td>U.S. Production to Sales (BLS)</td>
<td>1.34</td>
<td>0.57</td>
</tr>
</tbody>
</table>

Note: For details on the data sources, see the data appendix.
Table 22. Variance of Wage Differentials across Sectors and Locations

<table>
<thead>
<tr>
<th></th>
<th>Industry (ILO)</th>
<th>Occupation (ILO)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Location</td>
<td>Sector</td>
</tr>
<tr>
<td>World</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Proportion of variance</td>
<td>0.85</td>
<td>0.04</td>
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<tr>
<td>Observations</td>
<td>46</td>
<td>19</td>
</tr>
<tr>
<td>OECD</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Proportion of variance</td>
<td>0.84</td>
<td>0.06</td>
</tr>
<tr>
<td>Observations</td>
<td>27</td>
<td>12</td>
</tr>
<tr>
<td>LDC</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Proportion of variance</td>
<td>0.72</td>
<td>0.08</td>
</tr>
<tr>
<td>Observations</td>
<td>36</td>
<td>19</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Occupation (UBS)</th>
</tr>
</thead>
<tbody>
<tr>
<td>World</td>
<td></td>
</tr>
<tr>
<td>Proportion of variance</td>
<td>0.66</td>
</tr>
<tr>
<td>Observations</td>
<td>56</td>
</tr>
<tr>
<td>OECD</td>
<td></td>
</tr>
<tr>
<td>Proportion of variance</td>
<td>0.51</td>
</tr>
<tr>
<td>Observations</td>
<td>32</td>
</tr>
<tr>
<td>LDC</td>
<td></td>
</tr>
<tr>
<td>Proportion of variance</td>
<td>0.48</td>
</tr>
<tr>
<td>Observations</td>
<td>24</td>
</tr>
</tbody>
</table>

Notes: A panel has been selected such that the total number of observations is maximized
Table 23. Explanatory Power

<table>
<thead>
<tr>
<th>Panel: International cities, cross-border pairs</th>
<th>First quartile</th>
<th>Median</th>
<th>Third quartile</th>
</tr>
</thead>
<tbody>
<tr>
<td>CANADA-US</td>
<td>0.83</td>
<td>0.90</td>
<td>0.94</td>
</tr>
<tr>
<td>OECD</td>
<td>0.67</td>
<td>0.71</td>
<td>0.74</td>
</tr>
<tr>
<td>LDC</td>
<td>0.70</td>
<td>0.73</td>
<td>0.75</td>
</tr>
<tr>
<td>WORLD</td>
<td>0.69</td>
<td>0.72</td>
<td>0.75</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel: Intranational cities, no border</th>
<th>First quartile</th>
<th>Median</th>
<th>Third quartile</th>
</tr>
</thead>
<tbody>
<tr>
<td>CANADA-US</td>
<td>0.72</td>
<td>0.77</td>
<td>0.81</td>
</tr>
<tr>
<td>OECD</td>
<td>0.71</td>
<td>0.76</td>
<td>0.79</td>
</tr>
<tr>
<td>WORLD</td>
<td>0.70</td>
<td>0.75</td>
<td>0.79</td>
</tr>
<tr>
<td>LDC</td>
<td>0.70</td>
<td>0.73</td>
<td>0.75</td>
</tr>
</tbody>
</table>
Table 24. Variance of Prices across Locations

<table>
<thead>
<tr>
<th></th>
<th>Retail Prices</th>
<th>Wages Prices</th>
<th>Rental Prices</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A: International cities, cross-border pairs</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CANADA-US</td>
<td>0.07</td>
<td>0.13</td>
<td>0.61</td>
</tr>
<tr>
<td>OECD</td>
<td>0.25</td>
<td>0.17</td>
<td>3.05</td>
</tr>
<tr>
<td>LDC</td>
<td>0.42</td>
<td>0.59</td>
<td>11.18</td>
</tr>
<tr>
<td>WORLD</td>
<td>0.38</td>
<td>1.15</td>
<td>9.49</td>
</tr>
<tr>
<td><strong>Panel B: Intranational cities, no border</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CANADA-US</td>
<td>0.06</td>
<td>0.04</td>
<td>0.33</td>
</tr>
<tr>
<td>OECD</td>
<td>0.06</td>
<td>0.03</td>
<td>0.27</td>
</tr>
<tr>
<td>LDC</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>WORLD</td>
<td>0.07</td>
<td>0.03</td>
<td>0.28</td>
</tr>
</tbody>
</table>
Table 25. Variance Decomposition (median across EIU goods)

<table>
<thead>
<tr>
<th>Total Fraction of variance account for by:</th>
<th>Wages</th>
<th>Land Prices</th>
<th>Trade cost</th>
<th>Error</th>
<th>Covariance</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A: International cities, cross-border pairs</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CANADA-US</td>
<td>0.07</td>
<td>10.1</td>
<td>7.8</td>
<td>36.1</td>
<td>12.8</td>
</tr>
<tr>
<td>OECD</td>
<td>0.25</td>
<td>2.3</td>
<td>1.7</td>
<td>60.0</td>
<td>29.4</td>
</tr>
<tr>
<td>LDC</td>
<td>0.42</td>
<td>2.5</td>
<td>12.6</td>
<td>53.9</td>
<td>27.2</td>
</tr>
<tr>
<td>WORLD</td>
<td>0.38</td>
<td>7.7</td>
<td>3.4</td>
<td>50.7</td>
<td>28.1</td>
</tr>
<tr>
<td><strong>Panel B: Intrational cities, no border</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CANADA-US</td>
<td>0.06</td>
<td>4.8</td>
<td>5.9</td>
<td>53.8</td>
<td>24.8</td>
</tr>
<tr>
<td>OECD</td>
<td>0.06</td>
<td>5.1</td>
<td>3.7</td>
<td>51.4</td>
<td>26.5</td>
</tr>
<tr>
<td>LDC</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>WORLD</td>
<td>0.07</td>
<td>5.2</td>
<td>4.9</td>
<td>55.8</td>
<td>26.1</td>
</tr>
</tbody>
</table>
Table 26. Variance Decomposition (aggregate NIPA)

<table>
<thead>
<tr>
<th>Total</th>
<th>Fraction of variance account for by:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Wages</td>
</tr>
</tbody>
</table>

**Panel A: International cities, cross-border pairs**

<table>
<thead>
<tr>
<th></th>
<th>Wages</th>
<th>Land Prices</th>
<th>Trade cost</th>
<th>Error</th>
<th>Covariance</th>
</tr>
</thead>
<tbody>
<tr>
<td>CANADA-US</td>
<td>0.10</td>
<td>31.8</td>
<td>16.6</td>
<td>38.5</td>
<td>13.1</td>
</tr>
<tr>
<td>OECD</td>
<td>0.36</td>
<td>10.7</td>
<td>6.2</td>
<td>54.6</td>
<td>28.5</td>
</tr>
<tr>
<td>LDC</td>
<td>0.75</td>
<td>15.2</td>
<td>16.7</td>
<td>45.7</td>
<td>22.4</td>
</tr>
<tr>
<td>WORLD</td>
<td>0.66</td>
<td>36.4</td>
<td>6.8</td>
<td>36.0</td>
<td>20.9</td>
</tr>
</tbody>
</table>

**Panel B: Intranational cities, no border**

<table>
<thead>
<tr>
<th></th>
<th>Wages</th>
<th>Land Prices</th>
<th>Trade cost</th>
<th>Error</th>
<th>Covariance</th>
</tr>
</thead>
<tbody>
<tr>
<td>CANADA-US</td>
<td>0.12</td>
<td>10.0</td>
<td>8.5</td>
<td>57.2</td>
<td>24.3</td>
</tr>
<tr>
<td>OECD</td>
<td>0.13</td>
<td>6.6</td>
<td>11.0</td>
<td>56.0</td>
<td>26.3</td>
</tr>
<tr>
<td>LDC</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>WORLD</td>
<td>0.10</td>
<td>8.6</td>
<td>6.3</td>
<td>59.7</td>
<td>25.4</td>
</tr>
</tbody>
</table>
Figure 18. Kernel Density Estimates of Price Distributions

Notes: The solid lines are kernel density estimates of the distribution of $q_{ijk}$, time averaged LOP deviations over the period 1990-2005. The dashed lines are kernel density estimates of the distribution of $(q_{ijk,t} - q_{ijk})$, time series deviations from these long-run values. Each chart contains a different location and commodity grouping as indicated by the headers.
Figure 19. Variance Decomposition as a Function of Traded Input Share
Figure 20. Variance Decomposition as a Function of Non-traded Labor Input Share
Model Appendix

This appendix presents the function forms of the model, the first-order conditions and details for the model solution.

Function forms

\[ U(C_j, N_j) \equiv (1 - \theta) \log(C_j) + \theta \log L_j \]  
(IV.37)

\[ C_j = \left( \sum_i^M (\beta_i)^{\frac{1}{\varepsilon}} (C_{ij})^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{1}{\varepsilon-1}} \]  
(IV.38)

\[ P_j = \left( \sum_i \beta_i (P_{ij})^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}} \]  
(IV.39)

\[ Y_j = A_j N_j^m \]  
(IV.40)

\[ R_{ij} = (G_{ij})^{\alpha_i} \left( (B_j N_{ij}^s)_{\gamma_i} (K_{ij})^{1-\gamma_i} \right)^{1-\alpha_i} \]  
(IV.41)

Constraints

\[ L_j + N_j = 1 \]

\[ \sum_i P_{ij} C_{ij} = P_j C_j \]

\[ \sum_i P_{ij} C_{ij}^m \leq W_j^m N_j^m + \varphi H_j K_j \]

\[ \sum_i P_{ij} C_{ij}^s \leq W_j^s N_j^s + (1 - \varphi) H_j K_j \]
where $\varphi \in (0, 1)$ is the capital income share received by the manufacturer, $(1 - \varphi)$ is the capital income share received by the retailer, $H_j$ is the price of capital, and $K_j$ is the amount of capital.

**Consumer and producer problems**

$$\max_{C_j} \{ (1 - \theta) \log (C_j) + \theta \log L_j + \lambda_j [W^m_j (1 - L_j) + \varphi H_j K_j - P_j C_j] \} \quad \text{(IV.42)}$$

$$\max_{C_j} \{ (1 - \theta) \log (C_j) + \theta \log L_j + \lambda_j [W^s_j (1 - L_j) + (1 - \varphi) H_j K_j - P_j C_j] \} \quad \text{(IV.43)}$$

$$\max_{N^m_j} \{ Q_{j} A_{j} N^m_j - W^m_j N^m_j \} \quad \text{(IV.44)}$$

$$\max_{P_{ij}, N^s_j} \{ \left( B_{j} N^s_{ij} \right)^{\gamma_i} (K_{ij})^{1 - \gamma_i} \}^{1 - \alpha_i} - Q_{ij} G_{ij} - W^s_j N^s_{ij} - H_j K_{ij} \} \quad \text{(IV.45)}$$

**Efficiency conditions**

$$C^A_{ij} = \beta_i \left( \frac{P_{ij}}{P_j} \right)^{-\varepsilon} C^A_j \quad \text{(IV.46)}$$

$$C^A_j = \frac{W^A_j (1 - \theta)}{\theta} (1 - N^A_j) \quad \text{(IV.47)}$$

$$N^m_j = 1 - \theta - \frac{\varphi \theta H_j K_j}{W^m_j} \quad \text{(IV.48)}$$

$$N^s_j = 1 - \theta - \frac{(1 - \varphi) \theta H_j K_j}{W^s_j} \quad \text{(IV.49)}$$

$$L^m_j = \theta + \frac{\varphi \theta H_j K_j}{W^m_j} \quad \text{(IV.50)}$$

$$L^s_j = \theta + \frac{(1 - \varphi) \theta H_j K_j}{W^s_j} \quad \text{(IV.51)}$$
\[ N_{ij}^s = \frac{(1 - \alpha_i) \gamma_i Q_{ij}}{\alpha_i W_j^s} R_{ij} \left( \frac{B_j Q_{ij} (1 - \alpha_i) \gamma_i}{\alpha_i W_j^s} \right)^{(\alpha_i - 1) \gamma_i} \]  
\[ \times \left( \frac{Q_{ij} (1 - \alpha_i) (1 - \gamma_i)}{H_j} \right)^{(\alpha_i - 1)(1 - \gamma_i)} \]  
\[ G_{ij} = R_{ij} \left( \frac{B_j Q_{ij} (1 - \alpha_i) \gamma_i}{\alpha_i W_j^s} \right)^{(\alpha_i - 1) \gamma_i} \left( \frac{Q_{ij} (1 - \alpha_i) (1 - \gamma_i)}{H_j} \right)^{(\alpha_i - 1)(1 - \gamma_i)} \]  
\[ K_{ij} = \frac{(1 - \alpha_i) (1 - \gamma_i) Q_{ij}}{\alpha_i H_j} R_{ij} \left( \frac{B_j Q_{ij} (1 - \alpha_i) \gamma_i}{\alpha_i W_j^s} \right)^{(\alpha_i - 1) \gamma_i} \]  
\[ \times \left( \frac{Q_{ij} (1 - \alpha_i) (1 - \gamma_i)}{H_j} \right)^{(\alpha_i - 1)(1 - \gamma_i)} \]  
\[ Q_{jj} = M C_j = \frac{W_j^m}{A_j} \]  
\[ P_{ij} = M C_{ij}^s = \frac{(Q_{ij})^{\alpha_i}}{\alpha_i} \left( \frac{W_j^s}{\alpha_i} \gamma_i (H_j) (1 - \gamma_i) \right)^{(1 - \alpha_i)} \]  
\[ \times (1 - \alpha_i) (\gamma_i)^{(1 - \gamma_i)} (1 - \gamma_i)^{(1 - \alpha_i)} \]  
\[ \text{Price relationships} \]  
\[ Q_{ji} = (1 + \tau_{ji}) Q_{jj} \]  
\[ \text{The retail firm} \]  
\[ N_j^s = 1 - \theta - \frac{(1 - \varphi) \theta H_j K_j}{W_j^s} = \sum_i N_{ij}^s = \sum_i \left\{ \frac{(1 - \alpha_i) \gamma_i Q_{ij}}{\alpha_i W_j^s} R_{ij} \left( \frac{B_j Q_{ij} (1 - \alpha_i) \gamma_i}{\alpha_i W_j^s} \right)^{(\alpha_i - 1) \gamma_i} \right\} \]  
\[ \times \left( \frac{Q_{ij} (1 - \alpha_i) (1 - \gamma_i)}{H_j} \right)^{(\alpha_i - 1)(1 - \gamma_i)} \]  
\[ G_{ij} = R_{ij} \left( \frac{B_j Q_{ij} (1 - \alpha_i) \gamma_i}{\alpha_i W_j^s} \right)^{(\alpha_i - 1) \gamma_i} \left( \frac{Q_{ij} (1 - \alpha_i) (1 - \gamma_i)}{H_j} \right)^{(\alpha_i - 1)(1 - \gamma_i)} \]

164
General equilibrium

Manufacturing Labor Market

The labor supply of the manufacturer is used in the manufacturing process, which implies:

\[ \frac{Y_j}{A_j} = 1 - \theta - \frac{\varphi H_j K_j}{W^m_j} \]  \hspace{1cm} (IV.60)

Goods Market

In the global general equilibrium all the conditions of partial equilibrium must hold. However we also require that the supply of each good equals the demand for each good. This is where the treatment of trade costs becomes crucial. We will assume that trade costs are of the iceberg variety, so the physical resource constraint for good \( j \) must satisfy:

\[ Y_j = \sum_i G_{ji} (1 + \tau_{ij}) \]  \hspace{1cm} (IV.61)

In words: the units produced equal the demand of traded inputs of retailers at the destinations plus a fraction lost to iceberg costs. The aggregate fraction lost will depend on the equilibrium allocations since the loss along any bilateral trade route is proportional to the volume of trade along that branch:

\[ \frac{T_j}{Y_j} = \frac{\sum_i G_{ji} \tau_{ij}}{\sum_i G_{ji} (1 + \tau_{ij})} \]  \hspace{1cm} (IV.62)
Returning to our global equilibrium, we substitute the optimal traded input choices of the retailers into the resource constraint to arrive at:

\[ Y_j = \sum_i R_{ji} \left( \frac{B_i Q_{ji}}{W_i^m} \right) \left( \frac{1 - \alpha_j}{\alpha_j} \right)^{(\alpha_j-1)\gamma_j} \left( \frac{Q_{ji}}{H_i} \right) \left( \frac{1 - \alpha_j}{\alpha_j} \right)^{(\alpha_j-1)(1-\gamma_j)} (1 + \tau_{ji}) \]

Recall IV.60:

\[ \frac{Y_j}{A_j} = 1 - \theta - \frac{\varphi \theta H_j K_j}{W_j^m} \]

Combining these last two we get:

\[ \sum_i R_{ji} \left( \frac{B_i Q_{ji}}{W_i^s} \right) \left( \frac{1 - \alpha_j}{\alpha_j} \right)^{(\alpha_j-1)\gamma_j} \left( \frac{Q_{ji}}{H_i} \right) \left( \frac{1 - \alpha_j}{\alpha_j} \right)^{(\alpha_j-1)(1-\gamma_j)} (1 + \tau_{ji}) = A_j \left( 1 - \theta - \frac{\varphi \theta H_j K_j}{W_j^m} \right) \] (IV.63)

The equilibrium of the retailer implies:

\[ R_{ij} = C_{ij}^m + C_{ij}^s = \beta_i \left( \frac{P_{ij}}{P_j} \right)^{-\varepsilon} (C_{ij}^m + C_{ij}^s) \]

Assuming that \( \varepsilon = 1 \) (for the rest of the text), we have:

\[ R_{ij} = C_{ij}^m + C_{ij}^s = \frac{\beta_i}{P_{ij}} (P_j C_{ij}^m + P_j C_{ij}^s) = \frac{\beta_i}{P_{ij}} (N_j^m W_j^m + N_j^s W_j^s + H_j K_j) \]

\[ R_{ji} = \frac{\beta_j}{P_{ji}} (N_i^m W_i^m + N_i^s W_i^s + H_i K_i) \]

which says that the total income (sales) of the retailer from good \( i \) is equal to the share of that good in the budget of the region. Thus, we have

\[ \sum_i R_{ji} \left( \frac{B_i Q_{ji}}{W_i^s} \right) \left( \frac{1 - \alpha_j}{\alpha_j} \right)^{(\alpha_j-1)\gamma_j} \left( \frac{Q_{ji}}{H_i} \right) \left( \frac{1 - \alpha_j}{\alpha_j} \right)^{(\alpha_j-1)(1-\gamma_j)} (1 + \tau_{ji}) = A_j \left( 1 - \theta - \frac{\varphi \theta H_j K_j}{W_j^m} \right) \]
\[
\sum_i \frac{\beta_j}{Q_{ji}} (N_i^{m}W_i^{m} + N_i^{s}W_i^{s} + H_iK_i) \left( \frac{B_i Q_{ji} (1-\alpha_j) \gamma_j}{W_i^i} \right)^{(\alpha_j-1)\gamma_j} \\
\times \left( \frac{Q_{ji} (1-\alpha_j) (1-\gamma_j)}{\alpha_j} \right)^{(\alpha_j-1)(1-\gamma_j)} (1 + \tau_{ji}) \right) = A_j \left( 1 - \theta - \frac{\varphi \theta H_j K_j}{W_j^m} \right)
\]

Recall the price set by the retailer:

\[
P_{ij} = \frac{(Q_{ij})\alpha_i \left( \frac{W_i^s}{B_i} \right)^{\gamma_i} (H_j)^{(1-\gamma_i)}}{\alpha_i^{\alpha_i} \left( 1 - \alpha_i \right) (1 - \gamma_i)^{(1-\gamma_i)}} (1 - \alpha_i) (1 - \gamma_i)^{(1-\gamma_i)} (1 - \alpha_i) (1 - \gamma_i)
\]

which is to say:

\[
P_{ji} = \frac{(Q_{ji})\alpha_j \left( \frac{W_i^s}{B_i} \right)^{\gamma_j} (H_j)^{(1-\gamma_j)}}{\alpha_j^{\alpha_j} \left( 1 - \alpha_j \right) (1 - \gamma_j)^{(1-\gamma_j)}} (1 - \alpha_j) (1 - \gamma_j)^{(1-\gamma_j)} (1 - \alpha_j) (1 - \gamma_j)
\]

Thus,

\[
\sum_i \frac{\beta_j}{Q_{ji}} (N_i^{m}W_i^{m} + N_i^{s}W_i^{s} + H_iK_i) \left( \frac{B_i Q_{ji} (1-\alpha_j) \gamma_j}{W_i^i} \right)^{(\alpha_j-1)\gamma_j} \\
\times \left( \frac{Q_{ji} (1-\alpha_j) (1-\gamma_j)}{\alpha_j} \right)^{(\alpha_j-1)(1-\gamma_j)} (1 + \tau_{ji}) \right) = A_j \left( 1 - \theta - \frac{\varphi \theta H_j K_j}{W_j^m} \right)
\]

By using \(Q_{jj} = \frac{W_j^m}{A_j}\) and \(Q_{ji} = (1 + \tau_{ji}) Q_{jj}\), we can write

\[
\sum_i \frac{\beta_j}{Q_{ji}} (N_i^{m}W_i^{m} + N_i^{s}W_i^{s} + H_iK_i) \alpha_j (1 + \tau_{ji}) = A_j \left( 1 - \theta - \frac{\varphi \theta H_j K_j}{W_j^m} \right)
\]

This is the first equation for the relation between \(N^m W^m\), \(N^s W^s\), and \(H K\).
Retailing Labor Market

We have the following condition for the retailing labor market equilibrium:

\[
N_j^s = \sum_i N_{ij}^s = \sum_i \left\{ \frac{(1-\alpha_i)\gamma_i Q_{ij}}{\alpha_i W_j^m} R_{ij} \left( \frac{B_j Q_{ij} (1-\alpha_i)\gamma_i}{\alpha_i W_j^m} \right)^{(\alpha_i-1)\gamma_i} \right. \\
\left. \times \left( \frac{Q_{ij} (1-\alpha_i)(1-\gamma_i)}{\alpha_i} \right)^{(\alpha_i-1)(1-\gamma_i)} \right\}
\]

\[
N_j^s W_j^s = (N_j^m W_j^m + N_j^s W_j^s + H_j K_j) \sum_i (1 - \alpha_i) \gamma_i \beta_i \quad (IV.65)
\]

This is the second equation for the relation between \(N^m W^m\), \(N^s W^s\), and \(HK\).

Capital Market

We have the following condition for the capital market equilibrium:

\[
K_j = \sum_i K_{ij} = \sum_i \left\{ \frac{(1-\alpha_i)(1-\gamma_i) Q_{ij}}{\alpha_i H_j} R_{ij} \left( \frac{B_j Q_{ij} (1-\alpha_i)\gamma_i}{\alpha_i W_j^m} \right)^{(\alpha_i-1)\gamma_i} \right. \\
\left. \times \left( \frac{Q_{ij} (1-\alpha_i)(1-\gamma_i)}{\alpha_i} \right)^{(\alpha_i-1)(1-\gamma_i)} \right\}
\]

\[
H_j K_j = (N_j^m W_j^m + N_j^s W_j^s + H_j K_j) \sum_i (1 - \alpha_i) (1 - \gamma_i) \beta_i \quad (IV.66)
\]

This is the third equation for the relation between \(N^m W^m\), \(N^s W^s\), and \(HK\).

Implications for Wages, Rents, Wage Income, and Capital Income

Recall IV.64, IV.65, IV.66, which are:

\[
N_j^m W_j^m = \alpha_j \beta_j \sum_i (N_i^m W_i^m + N_i^s W_i^s + H_i K_i)
\]

\[
N_j^s W_j^s = (N_j^m W_j^m + N_j^s W_j^s + H_j K_j) \sum_i (1 - \alpha_i) \gamma_i \beta_i
\]
\[ H_jK_j = (N^m_jw^m_j + N^s_jw^s_j + H_jK_j) \sum_i (1 - \alpha_i)(1 - \gamma_i) \beta_i \]

Combine IV.65 and IV.66 to get:

\[
H_jK_j = \frac{\sum_i (1 - \alpha_i)(1 - \gamma_i) \beta_i}{\sum_i (1 - \alpha_i) \gamma_i \beta_i}
\]

and

\[
(H_jK_j + N^s_jw^s_j) = N^m_jw^m_j \frac{\sum_i (1 - \alpha_i) \beta_i}{(1 - \sum_i (1 - \alpha_i) \beta_i)}
\]

and thus

\[
\frac{N^s_jw^s_j}{N^m_jw^m_j} = \frac{\sum_i (1 - \alpha_i) \gamma_i \beta_i}{(1 - \sum_i (1 - \alpha_i) \beta_i)}
\]

which show that the sectoral wage incomes and capital incomes are all proportional to each other within each city.

Recall the individual optimality condition for the retailer:

\[ N^s_j = 1 - \theta - \frac{(1 - \varphi) \theta H_jK_j}{w^s_j} \]

\[ N^s_jw^s_j = w^s_j (1 - \theta) - (1 - \varphi) \theta H_jK_j \]

Combine this with IV.67 to get:

\[
H_jK_j = \frac{\left( N^s_jw^s_j \right) \sum_i (1 - \alpha_i)(1 - \gamma_i) \beta_i}{\sum_i (1 - \alpha_i) \gamma_i \beta_i}
\]

\[ N^s_jw^s_j \left( 1 + \frac{(1 - \varphi) \theta \sum_i (1 - \alpha_i)(1 - \gamma_i) \beta_i}{\sum_i (1 - \alpha_i) \gamma_i \beta_i} \right) = w^s_j (1 - \theta) \]

\[ N^s_j = \frac{(1 - \theta) \sum_i (1 - \alpha_i) \gamma_i \beta_i}{(\sum_i (1 - \alpha_i) \gamma_i \beta_i) + (1 - \varphi) \theta \sum_i (1 - \alpha_i)(1 - \gamma_i) \beta_i} \]

which shows that \( N^s_j \) is constant across regions. In a special case in which the share of capital is equal to zero in the retail production function (i.e., \( \gamma_i = 0 \)), or in which the share of capital income received by the retailer is equal to zero (i.e., \( \varphi = 1 \)), we have \( N^s_j = (1 - \theta) \).
Combine IV.69 with IV.64 to get:

\[
\frac{N^j_s W^s_j (1 - \sum_i (1 - \alpha_i) \beta_i)}{\sum_i (1 - \alpha_i) \gamma_i \beta_i} = \frac{N^m_j W^m_j}{N^m_k W^m_k} = \frac{\alpha_j \beta_j}{\alpha_k \beta_k} = \frac{N^s_j W^s_j}{N^s_k W^s_k}
\]

which show that the manufacturing wage income and the retailing wage income are proportional across cities. It is implied that:

\[
\frac{W^s_j}{W^s_k} = \frac{\alpha_j \beta_j}{\alpha_k \beta_k}
\]

(IV.71)

since \(N^s_j\) is constant across regions. If we also use IV.67, we obtain:

\[
\frac{H_j K_j}{H_k K_k} = \frac{\alpha_j \beta_j}{\alpha_k \beta_k}
\]

(IV.72)

where \(K_A\) is the capital stock in city \(A = j, k\).

Recall the individual optimality conditions for both the retailer and the manufacturer:

\[
W_j^m N_j^m = W_j^m (1 - \theta) - \varphi \theta H_j K_j
\]

\[
W_j^s N_j^s = W_j^s (1 - \theta) - (1 - \varphi) \theta H_j K_j
\]

These conditions can be combined to obtain:

\[
\frac{W_j^m}{W_j^s} = \frac{\varphi (1 - \theta) - \varphi N_j^s}{(1 - \varphi) (1 - \theta) - (1 - \varphi) N_j^m}
\]

Recall IV.69:

\[
\frac{N^m_j W^m_j}{N^s_j W^s_j} = \frac{(1 - \sum_i (1 - \alpha_i) \beta_i)}{\sum_i (1 - \alpha_i) \gamma_i \beta_i}
\]

170
Combine the last two expressions to get:

\[
\frac{N_j^m W_j^m}{N_j^s W_j^s} = \frac{N_j^m}{N_j^s} \frac{\varphi (1 - \theta) - \varphi N_j^s}{(1 - \varphi) (1 - \theta) - (1 - \varphi) N_j^m} = \frac{(1 - \sum_i (1 - \alpha_i) \beta_i)}{\sum_i (1 - \alpha_i) \gamma_i \beta_i}
\]

\[
N_j^m \left( \frac{\varphi (1 - \theta)}{N_j^s} - \varphi \right) \sum_i (1 - \alpha_i) \gamma_i \beta_i
\]

\[
= (1 - \varphi) (1 - \theta) \left( 1 - \sum_i (1 - \alpha_i) \beta_i \right) - (1 - \varphi) N_j^m \left( 1 - \sum_i (1 - \alpha_i) \beta_i \right)
\]

which can be combined with IV.70 (i.e., \(N_j^s\)) to obtain:

\[
N_j^m = (1 - \theta) - \frac{\varphi \theta (\phi_0 - \phi_1) (1 - \theta)}{\varphi \theta (\phi_0 - \phi_1) + (1 - \phi_0)}
\]

\[
= (1 - \theta) \text{ when } \varphi = 0 \text{ or } \gamma_i = 1
\]

where \(\phi_0 \equiv \sum_i (1 - \alpha_i) \beta_i, \phi_1 \equiv \sum_i (1 - \alpha_i) \gamma_i \beta_i, \phi_0 - \phi_1 = \sum_i (1 - \alpha_i) (1 - \gamma_i) \beta_i, \phi_2 \equiv \sum_i \alpha_i \beta_i = 1 - \phi_0.\) This shows that \(N_j^m\) is constant and equal across cities. The level of effort is equal to the leisure share of expenditure, \((1 - \theta)\) when either rental income is zero for the manufacturer \((\varphi = 0)\) or when retail production is labor-only \(\gamma_i = 1.\) Effort is declining in asset income.

\[
\frac{N_j^m}{(1 - \theta)} = 1 - \frac{\varphi \theta (\phi_0 - \phi_1)}{\varphi \theta (\phi_0 - \phi_1) + (1 - \phi_0)}
\]

\[
\frac{\partial N_j^m}{\partial \varphi} = -\theta (\phi_0 - \phi_1) d^{-1} + \varphi \theta (\phi_0 - \phi_1) d^{-2}
\]

\[
= \theta (\phi_0 - \phi_1) d^{-1} [-1 + \varphi d^{-1}]
\]

\[
\text{sign} \frac{\partial N_j^m}{\partial \varphi} = \text{sign} [-1 + \varphi d^{-1}] \text{ since } \theta (\phi_0 - \phi_1) d^{-1} > 0
\]

\[
\varphi < d
\]
\[ N_j^m = \frac{(1 - \phi_0)(1 - \theta)}{(1 - \phi_0) + \varphi \theta (\phi_0 - \phi_1)} \]

\[ N_j^s = \frac{\phi_1(1 - \theta)}{\phi_1 + (1 - \varphi) \theta (\phi_0 - \phi_1)} \]

\[ W_j^m = \frac{(1 - \phi_0) + \varphi \theta (\phi_0 - \phi_1)}{(1 - \varphi)(1 - \theta)(1 - \phi_0) + \varphi \theta (\phi_0 - \phi_1) - (1 - \varphi) \phi_1 + (1 - \varphi) \theta (\phi_0 - \phi_1)} \]

**Implications for Price Ratios across Cities**

Recall the retail price of good \( i \) in city \( j \) and city \( k \):

\[ P_{ij} = \frac{(Q_{ij})^{\alpha_i} \left( \frac{W_j}{B_j} \right)^{(1 - \alpha_i)\gamma_i} (H_j)^{(1 - \alpha_i)(1 - \gamma_i)}}{\alpha_i \gamma_i \left(1 - \alpha_i\right)(1 - \gamma_i)^3} \]

\[ P_{ik} = \frac{(Q_{ik})^{\alpha_i} \left( \frac{W_k}{B_k} \right)^{(1 - \alpha_i)\gamma_i} (H_k)^{(1 - \alpha_i)(1 - \gamma_i)}}{\alpha_i \gamma_i \left(1 - \alpha_i\right)(1 - \gamma_i)^3} \]

Take their ratio to get:

\[ \frac{P_{ij}}{P_{ik}} = \frac{(Q_{ij})^{\alpha_i} \left( \frac{W_j}{B_j} \right)^{(1 - \alpha_i)\gamma_i} (H_j)^{(1 - \alpha_i)(1 - \gamma_i)}}{(Q_{ik})^{\alpha_i} \left( \frac{W_k}{B_k} \right)^{(1 - \alpha_i)\gamma_i} (H_k)^{(1 - \alpha_i)(1 - \gamma_i)}} \]

By using \( Q_{ij} = (1 + \tau_{ij}) Q_{it} \), we can write the ratio of the price of good \( i \) across regions \( j \) and \( k \) as follows:

\[ \frac{P_{ij}}{P_{ik}} = \frac{((1 + \tau_{ij}))^{\alpha_i} \left( \frac{W_j}{B_j} \right)^{(1 - \alpha_i)\gamma_i} (H_j)^{(1 - \alpha_i)(1 - \gamma_i)}}{((1 + \tau_{ik}))^{\alpha_i} \left( \frac{W_k}{B_k} \right)^{(1 - \alpha_i)\gamma_i} (H_k)^{(1 - \alpha_i)(1 - \gamma_i)}} \]
By using IV.71 and IV.72, the analytical solution for the price ratios can be written as:

\[
\frac{P_{ij}}{P_{ik}} = \left( \frac{B_k}{B_j} \right)^{\gamma_i / \alpha_i} \left( \frac{\alpha_j \beta_j}{\alpha_k \beta_k} \right)^{1 - \alpha_i} \left( 1 + \tau_{ij} \right)^{\alpha_i} \left( \frac{K_k}{K_j} \right)^{(1 - \alpha_i)(1 - \gamma_i)}
\]

where \( K_j \) is the total amount of capital in city \( j \).

**Estimation Appendix**

The derivation of the variance decomposition of equation IV.33 can be written as follows:

\[
\text{var}_{jk} [E_t (q_{ijk,t})] = \text{var}_{jk} [(1 - \tilde{\alpha}_i) \tilde{\gamma}_i E_t (\omega_{jk,t})] + \text{var}_{jk} [(1 - \tilde{\alpha}_i)(1 - \tilde{\gamma}_i) E_t (h_{jk,t})] + \text{var}_{jk} [\tilde{\alpha}_i E_t (\tilde{I}_{ijk,t} \tilde{\delta}_i \tilde{d}_{jk})] + \text{var}_{jk} [E_t (\tilde{\varepsilon}_{ijk,t})] + 2 \text{cov} \left( (1 - \tilde{\alpha}_i) \tilde{\gamma}_i E_t (\omega_{jk,t}), (1 - \tilde{\alpha}_i) (1 - \tilde{\gamma}_i) E_t (h_{jk,t}) \right)
\]

\[
+ 2 \text{cov} \left( (1 - \tilde{\alpha}_i) \tilde{\gamma}_i E_t (\omega_{jk,t}), \tilde{\alpha}_i E_t (\tilde{I}_{ijk,t} \tilde{\delta}_i \tilde{d}_{jk}) \right)
\]

\[
+ 2 \text{cov} \left( (1 - \tilde{\alpha}_i)(1 - \tilde{\gamma}_i) E_t (h_{jk,t}), \tilde{\alpha}_i E_t (\tilde{I}_{ijk,t} \tilde{\delta}_i \tilde{d}_{jk}) \right)
\]

where \( \tilde{\alpha}_i \)'s, \( \tilde{\gamma}_i \)'s, \( \tilde{I}_{ijk} \)'s, \( \tilde{\delta}_i \)'s and \( \tilde{\varepsilon}_{ijk} \)'s are all estimated values for the relevant variables. Note that the covariance terms including \( E_t (\tilde{\varepsilon}_{ijk,t}) \) are equal to zero by OLS regression.


